Using GOAL to Study for Exam CS1!

Actuarial University

QUESTION 5 OF 5 | Question # | Go! | Prev | Next

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Flag problems for review, record notes, and email a professor for help.

Written Answer

This problem features a written answer component. Please write out your solution using your preferred method. When you are ready to review your work and score yourself, click the following button.

Reveal / Review Written Answer Scoring

Written answers must be graded if you wish to include them in your GOAL Score during Practice sessions.

Reveal answers and review to score yourself.

Information

The random variable X represents the number of claims in the coming year from a risk. The distribution of X depends on the fixed, but unknown, value of a parameter, λ. The conditional distribution of X given a λ is $Poisson(\lambda)$.

The prior distribution of λ is $Gamma(\alpha, \beta)$. A Bayesian credibility model provides that the posterior mean of λ can be expressed as

$$E[\lambda|x_1, x_2, \ldots, x_n] = Z\left[\frac{\sum_{i=1}^{n} x_i}{n}\right] + (1 - Z)\frac{\alpha}{\beta}, \text{ where } Z = \frac{n}{\beta + n}$$

with $x_1, x_2, \ldots, x_n$ being past observed values of X arising in each of the past n years. In the following questions, assume the actual number of claims arising each year from this risk are as follows:

Year	Number of Claims
1	203
2	216
3	197
4	195
5	234
6	240
7	220
8	243
9	220
10	225

Background information in the same format you'd see on your exam.

Question - Part A Difficulty: Mastery

View difficulty level.

[5 points] (i) Assume that the parameters of the prior Gamma distribution are $\alpha = 200$ and $\beta = 1$. Plot the credibility factor in successive years. Comment on the plot.

Question - Part B Difficulty: Mastery

[5 points] (ii) Obtain the credibility estimate of the number of claims in successive years based on the prior distribution for γ in (i). Plot the credibility estimate and the actual number of claims on the same plot. Comment on the plot.

Question - Part C Difficulty: Mastery

Exam style questions mirroring IFoA points system for easy scoring.

[6 points] (iii)Now suppose the parameters of the prior Gamma distribution are $\alpha = 200$ and $\beta = 5$. Plot the credibility factor in successive years and overlay it onto the plot in (i). Provide a commentary on the differences between the two plots.

Question - Part D Difficulty: Mastery

Not sure where to begin? Some questions feature "Help Me Start" to guide you in the right direction.

[6 points] (iii)Now suppose the parameters of the prior Gamma distribution are $\alpha = 200$ and $\beta = 5$. Based on these parameters, obtain the credibility estimate of the number of claims in successive years. Overlay the plot of the new credibility estimates on the plot in (ii).

Help Me Start

See the solution below!

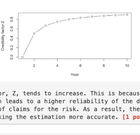
Question - Part A Difficulty: Mastery

[5 points] (i) Assume that the parameters of the prior Gamma distribution are $\alpha = 200$ and $\beta = 1$. Plot the credibility factor in successive years. Comment on the plot.

Self Assessment & Solution - Part A

Dive into the solutions and have each question broken down step by step.

```
n = 10
alpha = 200
beta = 1
Zx = numeric(n)
for (i in 1:n){
Zx[i] = (i-1)/(beta + (i-1))
}
Zx [2 points]

[1] 0.0000000 0.5000000 0.6666667 0.7500000 0.8000000 0.8333333 0.8571429
[8] 0.8750000 0.8888889 0.9000000

plot(x=1:n, Zx, type = "b", ylab = "Credibility factor Z", xlab = "Year") [2 points]
```




```
Over time, the credibility factor, Z, tends to increase. This is because more data
is gathered from the risk, which leads to a higher reliability of the data in
estimating the expected number of claims for the risk. As a result, the credibility
factor should also increase, making the estimation more accurate. [1 point]
```

Grade your performance using the self assesment.

Self Assessment Score: 5.00

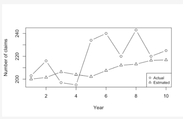
Question - Part B Difficulty: Mastery

[5 points] (ii) Obtain the credibility estimate of the number of claims in successive years based on the prior distribution for γ in (i). Plot the credibility estimate and the actual number of claims on the same plot. Comment on the plot.

Self Assessment & Solution - Part B

Dive into the solutions and have each question broken down step by step.

```
n = 10
alpha = 200
beta = 1
Zx = numeric(n)
PredNum = numeric(n)
ActNum<-c(203, 216, 197, 195, 234,240,220,243,220, 225)
PriorMean<-200
for (i in 1:n){
 Zx[i] = (i-1)/(beta + (i-1))
if (i==1) {meanObsData=0} else {meanObsData<-mean(ActNum[1:i-1])}
PredNum[i]<-Zx[i]*meanObsData+(1-Zx[i])*PriorMean
}
PredNum [2 points]

[1] 200.0000 201.5000 206.3333 204.0000 202.2000 207.5000 212.1429 213.1250
[9] 216.4444 216.8000

plot(x=1:n, ActNum, type = "b", ylab = "Number of claims", xlab = "Year") [2 points]
lines(x=1:n, PredNum,type = "b", pch = 2) [1 point]
legend("bottomrigh", legend=c("Actual", "Estmated"),pch=1:2, cex=0.8)
```




```
The initial estimate for the number of claims is 200, which is the mean of the prior
distribution of λ. The graph illustrates that the estimated number of claims
increases over time. This increase is due to the data from the risk itself being given
progressively more weight, i.e. credibility, while the collateral data, i.e. the prior
distribution of λ, is given correspondingly less weight. [1 point]
```

Grade your performance using the self assesment.

Self Assessment Score: 3.75

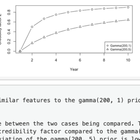
Question - Part C Difficulty: Mastery

[6 points] (iii)Now suppose the parameters of the prior Gamma distribution are $\alpha = 200$ and $\beta = 5$. Plot the credibility factor in successive years and overlay it onto the plot in (i). Provide a commentary on the differences between the two plots.

Self Assessment & Solution - Part C

Dive into the solutions and have each question broken down step by step.

```
n = 10
alpha = 200
beta = 1
Zx = numeric(n)
for (i in 1:n){
Zx[i] = (i-1)/(beta + (i-1))
}

Zx
[1] 0.0000000 0.5000000 0.6666667 0.7500000 0.8000000 0.8333333 0.8571429
[8] 0.8750000 0.8888889 0.9000000

n = 10
alpha = 200
beta = 5
Zx2 = numeric(n)
for (i in 1:n){
Zx2[i] = (i-1)/(beta + (i-1))
}

Zx2 [2 points]

[1] 0.0000000 0.1666667 0.2857143 0.3750000 0.4444444 0.5000000 0.5454545
[8] 0.5833333 0.6153846 0.6428571

plot(x=1:n, Zx, type = "b", ylab = "Credibility factor Z", xlab = "Year")
lines(x=1:n, Zx2,type = "b", pch = 2) [2 points]
legend("bottomrigh", legend=c("Gamma(200,1)", "Gamma(200,5)"),pch=1:2, cex=0.8)
```




```
The gamma(200, 5) prior shows similar features to the gamma(200, 1) prior, with an
increasing credibility factor.

There is a noticeable difference between the two cases being compared. The gamma(200, 5)
prior has a slower increase in credibility factor compared to the gamma(200, 1) prior.
This is because the standard deviation of the gamma(200, 5) prior is lower at 2.83, compared
to the gamma(200, 1) prior at 14.14.

The standard deviation of the prior distribution is an indication of how much confidence
is placed in the initial estimate of the number of claims. A smaller standard deviation
means that the initial estimate is believed to be more reliable. In Bayesian credibility,
the prior distribution represents the 'collateral data'. Therefore, a smaller standard
deviation means that the collateral data is considered to be more relevant.

In simpler terms, the smaller the standard deviation of the prior distribution, the more
relevant the collateral data is considered to be. This means that a smaller standard deviation
for the prior distribution would result in a lower credibility factor. This is because the
more relevant the collateral data, the lower the value of the credibility factor should be. [2 points]
```

Grade your performance using the self assesment.

Self Assessment Score: 6.00

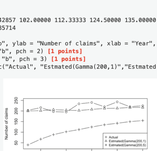
Question - Part D Difficulty: Mastery

[6 points] (iii)Now suppose the parameters of the prior Gamma distribution are $\alpha = 200$ and $\beta = 5$. Based on these parameters, obtain the credibility estimate of the number of claims in successive years. Overlay the plot of the new credibility estimates on the plot in (ii).

Self Assessment & Solution - Part D

Dive into the solutions and have each question broken down step by step.

```
n = 10
alpha = 200
beta = 1
beta2= 5
Zx = numeric(n)
PredNum = numeric(n)
PredNum2 = numeric(n)
ActNum<-c(203, 216, 197, 195, 234,240,220,243,220, 225)
PriorMean<-200
PriorMean2<-200/5
for (i in 1:n){
Zx[i] = (i-1)/(beta + (i-1))
Zx2[i] = (i-1)/(beta2 + (i-1))
if (i==1) {meanObsData=0} else {meanObsData<-mean(ActNum[1:i-1])}
PredNum[i]<-Zx[i]*meanObsData+(1-Zx[i])*PriorMean
PredNum2[i]<-Zx2[i]*meanObsData+(1-Zx2[i])*PriorMean2
}

PredNum2 [2 points]
[1]  40.00000  67.16667  88.42857 102.00000 112.33333 124.50000 135.00000
[8] 142.88333 149.84615 154.85714

plot(x=1:n, ActNum, type = "b", ylab = "Number of claims", xlab = "Year", ylim = c(30, 250)) [2 points]
lines(x=1:n, PredNum,type = "b", pch = 2) [1 points]
lines(x=1:n, PredNum2,type = "b", pch = 3) [1 points]
legend("bottomrigh", legend=c("Actual", "Estmated(Gamma(200,1)","Estmated(Gamma(200,5)"),pch=1:3, cex=0.
```



Grade your performance using the self assesment.

Self Assessment Score: 4.25

Rate this problem: Excellent | Needs Improvement | Inadequate

Rate a problem or give feedback.