



# Study Manual for Exam 9

14<sup>th</sup> Edition

Margaret Tiller Sherwood, FCAS, FSA,  
MAAA, FCA, CPCU, ARM, ERMP, CERA



A CAS Exam



*Actuarial & Financial Risk Resource Materials*  
Since 1972

Copyright © 2025, ACTEX Learning, a division of ArchiMedia Advantage Inc.

Printed in the United States of America.

No portion of this ACTEX Study Manual may be  
reproduced or transmitted in any part or by any means  
without the permission of the publisher.

## **TABLE OF CONTENTS**

### **A. Catastrophe and Reinsurance**

|    |                       |                                                                        |      |
|----|-----------------------|------------------------------------------------------------------------|------|
| 1. | Grossi & Kunreuther 2 | “An Introduction to Catastrophe Models and Insurance”                  | A1   |
| 2. | Grossi & Kunreuther 3 | “The Risk Assessment Process”                                          | A16  |
| 3. | Grossi & Kunreuther 4 | “Use of Catastrophe Models in Insurance Rate Making”                   | A28  |
| 4. | Grossi & Kunreuther 5 | “Sources, Nature, and Impact of Uncertainties on Catastrophe Modeling” | A35  |
| 5. | Grossi & Kunreuther 6 | “Insurance Portfolio Management”                                       | A46  |
| 6. | Clark                 | “Basics of Reinsurance Pricing”                                        | A52  |
| 7. | Bernegger             | “Swiss Re Exposure Curves and the MBBEFD Distribution Class”           | A128 |

### **B. Risk and Return**

|     |                       |                                                            |      |
|-----|-----------------------|------------------------------------------------------------|------|
| 8.  | Mildenhall & Major 3  | “Risk and Risk Measures”                                   | B1   |
| 9.  | Mildenhall & Major 4  | “Measuring Risk with Quantiles, VaR, and TVaR”             | B13  |
| 10. | Mildenhall & Major 5  | “Properties of Risk Measures and Advanced Topics”          | B23  |
| 11. | Mildenhall & Major 6  | “Risk Measures in Practice”                                | B33  |
| 12. | Mildenhall & Major 8  | “Classical Portfolio Pricing Theory,” Sections 8.5 and 8.7 | B46  |
| 13. | Mildenhall & Major 9  | “Classical Portfolio Pricing Practice”                     | B76  |
| 14. | Mildenhall & Major 10 | “Modern Portfolio Pricing Theory”                          | B84  |
| 15. | Mildenhall & Major 11 | “Modern Portfolio Pricing Practice”                        | B100 |

### **C. Financial Risk Management**

|     |                            |                                                                      |      |
|-----|----------------------------|----------------------------------------------------------------------|------|
| 16. | Coval, Jurek, and Stafford | “The Economics of Structured Finance”                                | C1   |
| 17. | Cummins Capital            | “Allocation of Capital in the Insurance Industry”                    | C22  |
| 18. | Cummins CAT                | “CAT Bonds and Other Risk-Linked Securities”                         | C51  |
| 19. | Mildenhall & Major 2       | “The Insurance Market and Our Case Studies”                          | C69  |
| 20. | Mildenhall & Major 8       | “Classical Portfolio Pricing Theory,” Sections 8.1-8.4, 8.6, and 8.9 | C75  |
| 21. | Mildenhall & Major 12      | “Classical Price Allocation Theory”                                  | C103 |
| 22. | Mildenhall & Major 13      | “Classical Price Allocation Practice”                                | C111 |
| 23. | Mildenhall & Major 14      | “Modern Price Allocation Theory”                                     | C115 |
| 24. | Mildenhall & Major 15      | “Modern Price Allocation Practice”                                   | C126 |
| 25. | Panning                    | “Managing Interest Rate Ris: ALM, Franchise Value, and Strategy”     | C132 |

### **D. Enterprise Risk Management**

|     |                |                                 |     |
|-----|----------------|---------------------------------|-----|
| 26. | Brehm et al. 1 | Introduction                    | D1  |
| 27. | Brehm et al. 2 | Application of Models of ERM    | D11 |
| 28. | Brehm et al. 3 | General Modeling Considerations | D50 |
| 29. | Brehm et al. 4 | Operational and Strategic Risk  | D71 |
| 30. | Brehm et al. 5 | Insurance Hazard Modeling       | D97 |

## NOTES

I have been updating this manual since 2015. There have been many syllabus changes since that time, including a major rearrangement of examination topics in the upper exams in 2024. For the current update, I reflected the 2025 syllabus material. This Study Guide is meant to be a supplement to the original material and not a substitute for it.

The material in Section D – Enterprise Risk Management originally was written by Victoria Grossack, FCAS for the CAS Exam 7 Study Manual for which it was originally prepared when that topic was covered on Exam 7. I thank her very much for allowing me to move that material to this manual.

When readings have been taken off the syllabus, I retained those questions from past exams that I think are relevant to the current readings, making changes to reflect the then-new syllabus material where appropriate. There is a section at the end of the Study Guide for questions applying to multiple readings that appeared on the 2018 and 2019 exams as the first two problems on each exam. As a practical matter, I suggest completing all other questions before answering questions that require material from multiple readings, regardless of where questions are placed in the exam.

Questions and parts of some solutions have been taken from material copyrighted by the Casualty Actuarial Society. They are reproduced in this study manual with the permission of the CAS solely to aid students studying for CAS exams. Students may also request past exams directly from the CAS or find them on the CAS website. I am very grateful to the CAS for its cooperation and permission to use this material. The CAS is not responsible for the structure or accuracy of this manual. In some cases, questions and answers have been edited or altered to be more accurate, reflect syllabus changes, or provide a better organized manual. Students should keep in mind that there may be more than one correct way to answer a question even if only one is shown.

Effective with the 2020 exams, the Casualty Actuarial Society no longer provides questions and answers for its exams, so the questions and answers from the 2020 and subsequent exams are not included in this update. Exam questions are identified by numbers in parentheses at the end of each question. Questions have four numbers separated by hyphens: the year of the exam, the number of the exam, the number of the question, and the points assigned. MC indicates that a multiple choice question has been converted into a true/false question. MTS in place of the second number indicates questions I added, with the first number denoting the year in which they were added. Review questions starting with R in Section D – Enterprise Risk management were added by Victoria.

Page numbers (p.) with solutions refer to the reading to which the question has been assigned unless otherwise noted.

I made a conscientious effort to eliminate mistakes and incorrect answers, but a few may remain. I am grateful to students who previously pointed out errors and encourage those who find others to bring them to my attention. If you find any errors, please submit them to [support@actexamdriver.com](mailto:support@actexamdriver.com). Please check the ACTEX Learning website for corrections subsequent to publication.

*Margaret Tiller Sherwood*  
FCAS, FSA, MAAA, FCA, CPCU, ARM, ERMP, CERA  
March 2025

**Patricia Grossi, Howard Kunreuther, and Don Windeler,  
Chapter 2: “An Introduction to Catastrophe Models and Insurance”  
in *Catastrophe Modeling: A New Approach to Managing Risk*,  
edited by Patricia Grossi and Howard Kunreuther, pp. 23–42**

**with March 21, 2021 Clarification and Errata  
by the CAS Syllabus Committee**

## OUTLINE

### I. OVERVIEW OF CATASTROPHE MODELS

#### A. History of Catastrophe Models

1. Some origins in property insurance – crude mapping of insured structures ends in 1960s
2. Other origins in seismological and meteorological measurements
  - a. Measurement of earthquakes begins with the first seismograph in 1800s
  - b. By 1970s studies published on sources and frequency of earthquakes, floods, and hurricanes
  - c. Late 1980s – mapping and measurement components combined to produce catastrophe modeling
3. Advances made in information technology and geographic information systems
  - a. Combine information on hazards with properties at risk
  - b. Hurricane Hugo (1989), Loma Prieta Earthquake (1989) and Hurricane Andrew (1992) promote the use of modeling
  - c. Hurricane Andrew results in nine insolvencies
4. Government recognition of the need for an accurate assessment of the impact of disasters leads to a FEMA report on earthquake estimation methodologies and the development of HAZUS, an open-source model

#### B. Structure of Catastrophe Models – Four Components

1. Hazard, e.g., for earthquakes, the epicenter, moment magnitude, etc.
2. Inventory – portfolio of properties
  - a. Geocoding assigns geographic coordinates
  - b. Other features included
    - 1) Construction type, stories, age
    - 2) Deductibles, coverage limits
3. Vulnerability – susceptibility to damage

4. Loss – quantification of the physical impact of natural hazard phenomenon
  - a. Varies from model to model
  - b. Loss to inventory
    - 1) **Direct losses** – “cost to repair and/or replace a structure”
    - 2) **Indirect losses** – “business interruption impacts and relocation costs of residents forced to evacuate their homes”
- C. Uses of a Catastrophe Model for Risk Management
  1. Model output quantified and presented to stakeholders
  2. Assessment of alternative risk management strategies
    - a. Mitigation
    - b. Insurance
    - c. Reinsurance
    - d. Catastrophe bonds
  3. Examples of outputs
    - a. GIS map
      - 1) Spatial representation of the number of displaced households
      - 2) Useful for emergency response officials
    - b. Exceedance probability curve
      - 1) **Exceedance probability curve** – “graphical representation of the probability that a certain level of loss will be surpassed in a given time period”
      - 2) Enables insurer to assess whether risk of the current loss profile is acceptable

## II. DERIVATION AND USE OF AN OCCURRENCE EXCEEDANCE PROBABILITY CURVE

### A. Definitions

1. Occurrence Exceedance Probability (OEP) – Probability that at least one loss exceeds the specified amount
2.
  - a. This is the distribution of the largest loss in the period
  - b. Based on the theory of order statistics.
3. Aggregate Exceedance Probability (AEP) – Probability that the sum of all losses during a given period exceeds some amount
4. Conditional Exceedance Probability (CEP) – Probability that the amount of a single event exceeds a specified loss amount. The CEP is equal to 1-CDF of the severity curve used by actuaries in other contexts

B. Generating an Occurrence Exceedance Probability Curve

1. Expected loss for a given event

$$E[L] = p_i L_i, \text{ where}$$

$p_i$  – probability of the occurrence of event  $E_i$

$L_i$  – loss associated with event  $E_i$

2. Average Loss Event

$$ALE = \sum_i p_i L_i$$

3. Occurrence exceedance probability of event  $E_i$  if only one of each event occurs in a year

$$OEP(L_i) = 1 - \prod_{j=1}^{i-1} (1 - p_j)$$

The resulting OEP is the probability that at least one loss exceeds a given value.

4. **Probable maximum loss (PML)** – “subjective risk metric . . . associated with a given probability of exceedance specified by the insurer”
  - a. Loss amount corresponding to a selected acceptable probability level
  - b. Alternatively, loss amount corresponding to a return period
  - c. Return period is the inverse of the annual probability of exceedance

C. Stakeholders and the Occurrence Exceedance Probability Curve

1. If liability split among several stakeholders, can develop OEP curves for each party
2. Various expected losses possess inherent uncertainty
3. CV on event loss generally decreases with the size of loss

III. INSURABILITY OF CATASTROPHE RISKS

A. Overview

1. Insurance a principal mechanism for managing risk and may be compulsory for mortgages and car licensing
2. Insurance pricing can signal riskiness of an activity, e.g., higher prices for younger drivers and smokers
3. But regulation may prevent insurance prices from fully reflecting risks
4. Question of how to allocate catastrophe risks in a similar way to the allocation of noncatastrophe risks
  - a. Sufficient data allows use of actuarial-based models for automobile coverages
  - b. But lack of data means catastrophe models must be used for natural disasters

B. Conditions for Insurability of a Risk

1. Two conditions for the insurability of a risk

- a. Able to estimate probability of an event's occurrence and extent of likely losses
  - 1) Base on past data or modeling and expert assessment of a risk
  - 2) Use data to create an exceedance probability
- b. Able to set premiums for each potential customer or class of customers
  - 1) If considerable uncertainty about a risk, higher premium may result
  - 2) If reduced capacity because of recent losses, industry may charge higher prices, especially if demand increases
2. Meeting the conditions for insurability does not imply profitability – may not be able to set a rate for which there is sufficient demand and cover costs and profit
3. Factors affecting the rate level decision
  - a. State regulation
  - b. Competition
  - c. Uncertainty of losses
  - d. Correlation of losses
  - e. **Adverse selection** – situation “when the insurer cannot distinguish (or does not discriminate through price) between the expected losses for different categories of risk, while the insured, possessing information unknown to the insurer, selects a price/coverage option more favorable to the insured”
  - f. **Moral hazard** – increased expected loss caused by the policyholder's behavior
  - g. Adverse selection and moral hazard not major problems for natural hazard risks

C. Uncertainty of Losses

1. Natural disaster hazards involve uncertain and potentially high losses
2. For such hazards, median loss is low but maximum loss is very high

D. Highly Correlated Losses

1. Simultaneous occurrence of many losses from a single event
2. Insurance markets do best when they have many policies whose potential losses are spatially and otherwise independent
3. Application of the law of large numbers

E. Determining Whether to Provide Coverage

1. Stone's two constraints
  - a. Survival constraint – insurers seek to maximize profits while meeting constraint
  - b. Stability constraint – less applicable in the case of catastrophe risks
2. Following 1989 disasters insurers focus on survival constraint
  - a. FL – insurers only sell wind damage coverage because required to do so; state insurance pools enable them to limit risk
  - b. CA – California Earthquake Authority makes earthquake coverage available



3. Meeting the survival constraint
  - a. Select portfolio with insolvency probability less than a selected probability
  - b. Number of policies needed to satisfy the constraint  

$$\Pr[\text{Total Loss} > (nz + A)] < p_1, \text{ where}$$

n - maximum number of policies that can be written  
 z - insurance premium  
 A – current surplus  
 p<sub>1</sub> – selected insolvency probability
  - c. Whether risk is insurable depends on whether cost of marketing and policy issuance is sufficiently low
  - d. Ways of meeting the constraint
    - 1) Increase the premium amount – will increase number of possible policies but will reduce the demand
    - 2) Reduce the coverage
    - 3) Transfer some of risk to others

#### IV. FRAMEWORK TO INTEGRATE RISK ASSESSMENT WITH RISK MANAGEMENT

##### A. Framework

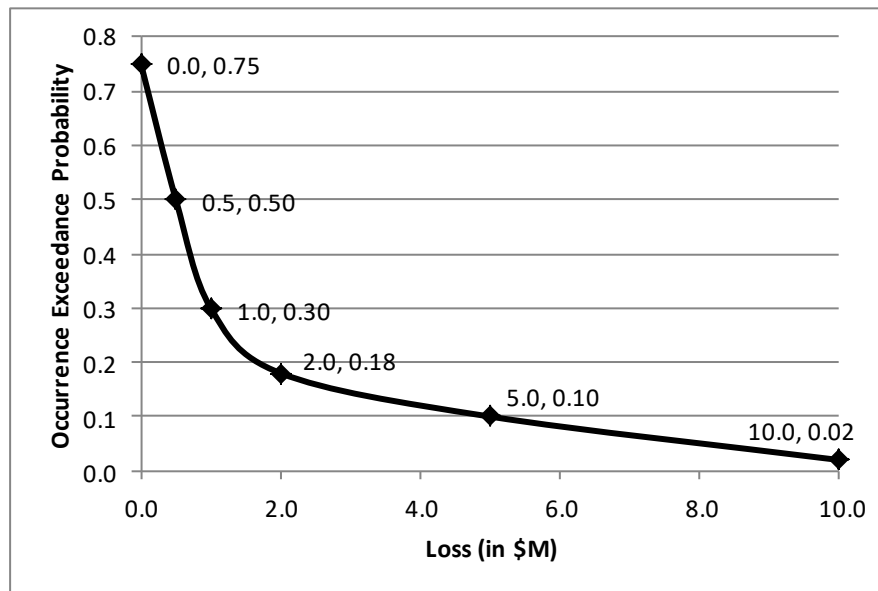
1. Assessment of risk using catastrophe modeling involving four modules
2. Stakeholders' decision processes produce risk management strategies
3. Goal is to maximize expected profits subject to meeting survival constraint

##### B. Risk Management Strategies

1. Risk reduction measures
  - a. Mitigation
  - b. Well-enforced building codes
  - c. Land-use regulations
2. Risk transfer
  - a. Insurance
  - b. Reinsurance
  - c. Catastrophe bonds
3. Strategies for insurers to reduce risk
  - a. Charge higher rates to reflect uncertainty
  - b. Change portfolio
  - c. Transfer risk

**PAST CAS EXAMINATION QUESTIONS**

1. The following occurrence exceedance probability curve is available for an insurer's portfolio:



- Briefly explain what an occurrence exceedance probability curve represents.
- The insurer wants to hold capital to support a 1-in-25 year Probable Maximum Loss (PML). Determine the loss level associated with this PML implied by the occurrence exceedance probability curve above.
- Briefly discuss three common uses for occurrence exceedance probability curves.

(12–8–9–1.5 Modified)

1.
  - a. An occurrence exceedance probability curve shows all possible levels of annual loss and the annual probability that the given loss level is exceeded.  
  
The probability for a loss to exceed a certain level given a period of time.
  - b.  $1/25 = .04 \Rightarrow \text{PML} = 10\text{M} - [((0.04 - 0.02)/(0.1 - 0.02)) * 5\text{M}] = 8.75\text{M}$
  - c. (Need three)
    - Calculate the PML for a given payout period
    - Calculate if the portfolio meets solvency goal
    - Decide how much proportion of the risks should be ceded
    - Used to calculate average annual loss
    - Set level of conservativeness like PML in 1/X chance
    - Used to find a strategy to change the portfolio if it is currently above the level of conservativeness
    - Used by emergency response unit to determine where damage might be and build strategies in times of catastrophes
    - Used when running logic trees. Instead of using point estimates, each branch of tree can have its own exceedance probability curve for the different outcomes. Then can combine.
    - Emergency management services: to evaluate potential risk of some regions and plan evacuation.
    - Reinsurance Broker: to evaluate the level of risk of their portfolio and estimate the impact of accepting new risks.

2. The following Occurrence Exceedance Probability curve is available for an insurance company's portfolio:

| Return Period | Occurrence Exceedance Probability | Loss          |
|---------------|-----------------------------------|---------------|
| 10,000        | 0.0001                            | \$200,000,000 |
| 500           | 0.0020                            | \$50,000,000  |
| 200           | 0.0050                            | \$20,000,000  |
| 100           | 0.0100                            | \$12,000,000  |
| 50            | 0.0200                            | \$7,000,000   |
| 33            | 0.0300                            | \$3,500,000   |
| 25            | 0.0400                            | \$1,500,000   |
| 20            | 0.0500                            | \$500,000     |

- a. The insurer specifies that its acceptable risk level is 1-in-250 year PML. Define PML and calculate the 1-in-250 year PML.
- b. The insurer decides to buy property catastrophe reinsurance protection up to the 1-in-500 year PML in the following treaties:

Quota share, where 30% is ceded up to a \$40 million loss limit, which inures to the benefit of the following:

- 100% placed 1<sup>st</sup> layer property catastrophe excess of loss treaty \$6 million xs \$4 million
- 90% placed 2<sup>nd</sup> layer property catastrophe excess of loss treaty \$10 million xs \$10 million
- 75% placed 3<sup>rd</sup> layer property catastrophe excess of Loss treaty \$30 million xs \$20 million

During the treaty year, the insurer suffers a \$45 million earthquake loss.

Calculate the amount of loss ceded to each of the reinsurance treaties and the net retained loss by the primary insurer.

(13-8-24-1/1.25)

3. Define the following terms:

- Occurrence Exceedance Probability
- Aggregate Exceedance Probability
- Conditional Exceedance Probability

(14-8-MTS-0.5/0.5/0.5)

2. a. PML = Probable Maximum Loss – largest loss likely to occur for the insurer

$$1/250 = 0.004$$

Must interpolate between 0.002 and 0.005

$$\frac{(0.004 - 0.002)}{(0.005 - 0.002)} \times (20 - 50) = -20$$

$$50,000,000 - 20,000,000 = 30,000,000 \text{ is } 1/250 \text{ PML}$$

- b. Ceded to Quota Share =  $0.3 \times 40\text{M} = 12\text{M}$   
Retained from QS =  $45 - 12 = 33\text{M}$

Ceded to first layer =  $100\% \times 6\text{M} = 6\text{M}$  (retained 4M)

Ceded to second layer =  $90\% \times 10\text{M} = 9\text{M}$  (retained 1M)

Ceded to third layer =  $75\% \times (33 - 20)\text{M} = 9.75\text{M}$  (retained 3.25M)

$$\text{Net retained} = 4\text{M} + 1\text{M} + 3.25\text{M} = 8.25\text{M}$$

3. Occurrence Exceedance Probability – The probability that at least one loss exceeds the specified loss amount.

Aggregate Exceedance Probability – The probability that the sum of all losses during a given period exceeds some amount.

Conditional Exceedance Probability – The probability that the amount on a single event exceeds a specified loss amount.

4. An insurance company is exposed to three independent catastrophic risks in three different regions in a given year. More than one event can occur in a year but each event can only occur once in a year. Events have the following size and probability:

| Event | Loss Amount  | Annual Probability of Occurrence |
|-------|--------------|----------------------------------|
| 1     | \$10,000,000 | 0.10                             |
| 2     | \$15,000,000 | 0.05                             |
| 3     | \$35,000,000 | 0.02                             |

- Calculate the Aggregate Exceedance Probabilities associated with the insurance company's exposure.
- Using a randomly generated number of 0.86, simulate the insured total loss.

(15-8-22-2.25/0.5)

4. a.

| Aggregate loss | Event combination | Probability of combination         | Exceedance probability    | F(x)   |
|----------------|-------------------|------------------------------------|---------------------------|--------|
| 60 M           | 1,2,3             | $(0.1)(0.05)(0.02) = 0.0001$       | 0                         | 1      |
| 50 M           | 2,3               | $(1-0.1)(0.05)(0.02) = 0.0009$     | 0.0001                    | 0.9999 |
| 45 M           | 1,3               | $(0.1)(1-0.05)(0.02) = 0.0019$     | $0.0001 + 0.0009 = 0.001$ | 0.999  |
| 35 M           | 3                 | $(1-0.1)(1-0.05)(0.02) = 0.0171$   | $0.001 + 0.0019 = 0.0029$ | 0.9971 |
| 25 M           | 1,2               | $(0.1)(0.05)(1-0.02) = 0.0049$     | $0.0029 + 0.0171 = 0.02$  | 0.98   |
| 15 M           | 2                 | $(1-0.1)(0.05)(1-0.02) = 0.0441$   | $0.02 + 0.0049 = 0.0249$  | 0.9751 |
| 10 M           | 1                 | $(0.1)(1-0.05)(1-0.02) = 0.0931$   | $0.0249 + 0.0441 = 0.069$ | 0.931  |
| 0 M            | -                 | $(1-0.1)(1-0.05)(1-0.02) = 0.8379$ | $0.069 + 0.0931 = 0.1621$ | 0.8379 |

b. 0.86 lays between 0.8379 and 0.931 on the F(x) distribution. Therefore, 10 M loss occurs.

5. In order for an insurance company to increase its return on capital, two reinsurance options are being considered:

I. \$5 million excess of \$5 million per risk:

- ALAE pro-rata
- Rate = 18% of premium

II. 20% Quota Share:

- Ceding commission of 30%
- Maximum ceded loss ratio of 150%

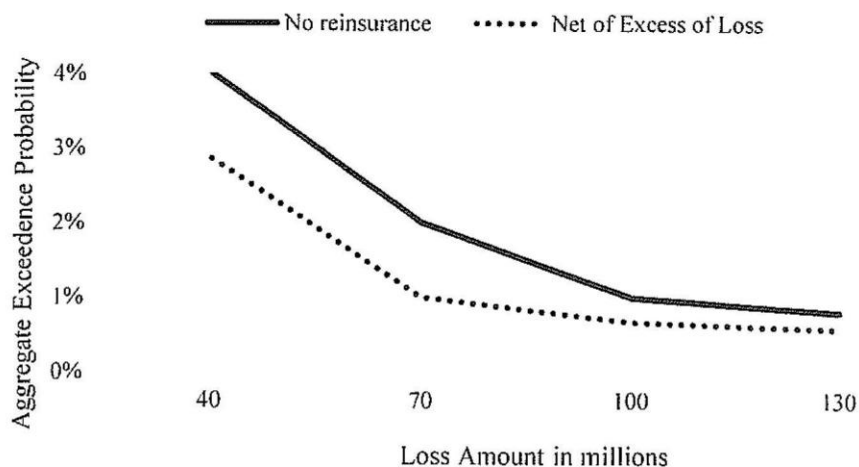
The insurance company must hold capital to support a 1-in-100 year Probable Maximum Loss. Additionally, the following information regarding the insurance company's performance last year is given:

- Premium: \$50,000,000
- Expenses: \$15,000,000
- Total Loss & ALAE: \$30,000,000
- Return on Capital: 5%

**Claims greater than \$5,000,000**

|         | Loss         | ALAE        |
|---------|--------------|-------------|
| Claim 1 | \$7,500,000  | \$1,500,000 |
| Claim 2 | \$10,000,000 | \$500,000   |

The Aggregate Exceedance Probability curve for the insurance company is shown below:



Determine the impact each reinsurance option would have had on last year's return on capital. Ignore investment income and taxes.

(16-8-19-3)



5. Under XOL:  
 Ceded Loss:  
 Claim 1 = 2.5M  
 Claim 2 = 5M  
 Total = 7.5M

Ceded ALAE:  
 Claim 1 =  $(2.5/7.5) * 0.5 = 0.5M$   
 Claim 2 = 0.25M Total = 0.75M

1% AEP w/ XOL - Company must hold \$70M based on curve.  
 Under QS, capital held is  $\$100M - (\$50M) * (0.2) * (1.5) = \$85M$

Ceded Premium under XOL =  $0.18 * 50M = 9M$   
 Ceded Premium under QS =  $0.2 * (50M) * (1 - 0.3) = 7M$

Retained Premium (net of commission) under XOL ( $\$50M - \$9M$ ) = \$41M  
 Expenses = \$15M  
 Retained Loss & ALAE =  $\$30M - \$7.5M - 0.75M = \$21.75M$   
 Profit =  $\$41M - \$15M - \$21.75M = \$4.25M$   
 ROE =  $\$4.25M / \$70M = 6.07\%$   
 Impact on ROE =  $6.07 - 5 = 1.07\%$

Retained Premium (net of commission) under QS ( $\$50M - \$7M$ ) = \$43M  
 Expenses = \$15M  
 Retained Loss & ALAE =  $\$30M * (.8) = \$24M$   
 Profit =  $\$43M - \$15M - \$24M = \$4M$   
 ROE =  $\$4M / \$85M = 4.71\%$   
 Impact on ROE =  $4.71 - 5 = -0.3\%$

6. A reinsurance company is evaluating whether or not to write a \$50 million excess of \$50 million catastrophe reinsurance contract with a primary insurer. The reinsurer is currently holding \$850 million of capital and is required to hold enough capital to survive a 1-in-250 event. Without the new contract, the reinsurance company has a 1-in-250 probable maximum loss (PML) of \$825 million which is solely driven by the hurricane peril.

Given the following:

- The primary insurer's PMLs are driven by the hurricane and earthquake perils only.
- The primary insurer's per occurrence annual PMLs by return period are as follows:

| Return Period (years) | PML (\$000,000) |
|-----------------------|-----------------|
| 1000                  | 125             |
| 500                   | 105             |
| 200                   | 95              |
| 100                   | 70              |
| 50                    | 50              |
| 25                    | 30              |
| 20                    | 25              |
| 10                    | 20              |
| 5                     | 15              |

- The largest hurricane event in the primary insurer's event catalog is \$45,000,000.
  - Calculate the ceded, and net, 1-in-250 PMLs for this contract for the primary insurer.
  - Evaluate whether the reinsurer should participate in this treaty.

(18-8-16-1.0/0.75)

6. a.

| Return Period (RP) | Exceedance Probability (EP) | Gross PML | Ceded PML | Net PML |
|--------------------|-----------------------------|-----------|-----------|---------|
| 500                | 0.002                       | 105       | 50        | 55      |
| 200                | 0.005                       | 95        | 45        | 50      |

Interpolate Gross PML for 250 between (.002, 105) and (.005, 95)

$$95 + (.005 - .004) / (.005 - .002) = 98.33M$$

$$\text{Ceded PML} = 98.33 - 50 = 48.33$$

$$\text{Net PML} = 50 M$$

- b. The reinsurer's 1-in-250 PML is driven completely by hurricane, so it should participate in the treaty to diversify the perils it is exposed to. Since the largest hurricane event for the primary insurer is 45m, the reinsurer will not increase its exposure to hurricane – only earthquake. This means the reinsurer's 1:250 should not grow by taking on additional EQ exposure. This is all, of course, on a modeling basis. There is potential for the model to be wrong, but the reinsurer should diversify.

Mehrdad Mahdyiar and Beverly Porter,

**Chapter 3: “The Risk Assessment Process: The Role of Catastrophe Modeling in Dealing with Natural Hazards” in *Catastrophe Modeling: A New Approach to Managing Risk*, edited by Patricia Grossi and Howard Kunreuther, pp. 45–68**

OUTLINE

I. INTRODUCTION

A. Probabilistic Approaches

1. Used by structural engineers in building design
2. Extended to estimate economic and insured losses resulting from catastrophes
3. For catastrophe losses, modeling preferable to actuarial techniques
  - a. Limited historical data
  - b. Changes in many inputs
    - 1) Property values, repair, and replacement costs
    - 2) Building materials, practices, and codes

B. Catastrophe Models

1. Complexity involved
  - a. Modeling of complex physical phenomena in time and space
  - b. Constructing detailed databases of building inventories
  - c. Estimating physical damage
  - d. Converting physical damage to monetary loss
  - e. Summing over a portfolio
2. Two components make up probabilistic risk analysis
  - a. **Hazard module** – module that “estimates the probability that the physical parameters that define the hazard will exceed various levels”
  - b. **Vulnerability module** – module that “estimates the probability that building damage will exceed various levels”
3. **Loss module** – module that “translates physical damage into monetary loss and estimates the probability of exceeding various levels of loss”
  - a. Produces exceedance probability
  - b. Used for financial analysis
4. Requirements for such models
  - a. Substantial amounts of data
  - b. Understanding of underlying physical occurrences

## II. HAZARD MODULE

### A. Overview

1. Source parameters needed
  - a. Most likely locations
  - b. Frequency of occurrences
  - c. Severity of occurrences
2. Development of probability distributions using the following:
  - a. Historical data
  - b. Statistical techniques
  - c. Understanding of natural hazards
3. Using source parameters, can estimate intensity for individual locations

### B. Locations of Potential Future Events

1. Earthquakes
  - a. Definition of the boundaries of a model domain based on the following:
    - 1) Important faults and seismic source zones
    - 2) Region's geological features
    - 3) Understanding of wave propagation physics
  - b. Identification of faults within the domain
    - 1) Information used
      - a) Records of historical seismicity
      - b) Fault trenching
      - c) Subsurface sounding techniques
      - d) Aerial photography
    - 2) Exclusion of identified faults if no activity for at least 10,000 years
  - c. Modeling based on source zones, not on faults
    - 1) Use spatial distribution of past earthquakes to estimate future ones
    - 2) Also simulate earthquakes where they have not occurred previously by smoothing historical data
    - 3) If little or no historical activity, use larger zones of background seismicity in combination with faults and source zones
  - d. Use paleoseismic data
    - 1) Because of long recurrence intervals, historical record may not identify all seismic sources
    - 2) **Paleoseismology** – “study of prehistoric earthquakes, particularly their location, timing, and size”

- 3) Steps in the estimation process
    - a) Locate sites showing evidence of such
    - b) Estimate total affected area
    - c) Convert area to a magnitude
  - 4) Considerable uncertainty in the process
  - e. Use geodetic survey data to identify regions under strain and with high earthquake potential
2. Hurricanes
- a. Most likely between 5 and 20 degrees latitude
  - b. By defining various parameters, can quantify geographical distributions
    - 1) Storm tracks
    - 2) Landfall location
    - 3) Track angle at landfall
  - c. Can also use more complex techniques, e.g., physically based numerical weather prediction models
  - d. Use historical track data to generate probability matrices, using smoothing techniques

C. Frequency of Occurrence

- 1. Overview
  - a. Frequency is the most critical and uncertain aspect of modeling
  - b. Uncertainty attributable to the following:
    - 1) Scarcity of historical data
    - 2) Lack of full understanding of physical mechanisms
- 2. Earthquakes
  - a. Large magnitude earthquakes have a mean recurrence interval of about 150 years
  - b. Stress history of a fault needed to assess its present condition and rupture potential
  - c. Modeling of the relationship between frequency and magnitude based on two factors
    - 1) **Characteristic earthquakes** – earthquakes of a similar magnitude produced when a fault or fault segment ruptures regularly
      - a) Generally not that predictable
      - b) Concept can estimate fault's accumulated strain and release
      - c) Identify by one magnitude or a range

## 2) Gutenberg-Richter relationship

$$\log(N) = a - bM, \text{ where}$$

$N$  – cumulative annual frequency

$a$  – reference magnitude

$b$  – “rate at which the log of the cumulative annual frequency of earthquakes decreases as the magnitude increases”

- a) Establish lower and upper bound magnitudes
- b) Parameters vary by region
- c) Estimate parameters from historical data and physical parameters
- d) Selection of upper bound magnitude is crucial

## 3) Combine all earthquake-related data for a source zone to produce seismic hazard maps showing ground motions that have a specific exceedance probability over 50 years

## 3. Hurricanes

## a. Necessary conditions for occurrence

- 1) Large expanse of warm ocean water – most active months are August and September in the North and January and February in the South
- 2) Relative absence of vertical shear – such increases with distance from tropical latitudes

b. **Vertical shear** – “winds that change appreciably in either magnitude or direction with height”D. Parameterizing Severity at the Hazard’s Sources

## 1. Model generates primary characteristics of simulated events in a source zone

## 2. Hurricane parameters

- a. Central barometric pressure
- b. Forward or translational speed
- c. Radius of maximum winds
- d. Track angle at land

## 3. Earthquake parameters

- a. Magnitude
- b. Focal depth
- c. Fault-rupture characteristics

## 4. Models impose limiting magnitude (upper) for earthquakes and limiting central pressure (lower) for hurricanes based on the following:

- a. Historical data
- b. Physical possibilities

E. Parameters for Local Intensity and Site Effects

## 1. Earthquakes

- a. Seismic waves emanate from the source throughout the region
- b. Factors affecting damage to structures
  - 1) Amplitude and frequency of waves
  - 2) Intervening geological materials
  - 3) Local soil materials at a site
- c. Attenuation equation
$$Y = F(f, M, r, \text{Source}, \text{Site}), \text{ where}$$
  - Y – ground motion amplitude
  - f – frequency
  - M – earthquake magnitude
  - r – source-to-site distance
  - Source – source rupture mechanisms
  - Site – local site effects on ground motion
- d. Equations are region-specific
- e. More ground motion produced by earthquakes with thrust and reverse faulting mechanisms than ones with strike-slip and normal faulting mechanisms
- f. Soft soil materials affect low-frequency ground motion whereas shallow soil materials affect high-frequency ground motion

## 2. Hurricanes

- a. Local windfields estimates based on a maximum over-water speed adjusted for the following:
  - 1) Storm asymmetry
  - 2) **Filling** – “rate at which central pressure increases as the storm moves inland”
  - 3) Local surface terrain
    - a) Lower speeds at lower elevations reflecting the following:
      - i) Horizontal drag force of the earth’s surface
      - ii) Obstacles, e.g., buildings
    - b) The rougher the surface, the greater the dissipation
- d. Wind duration affects local density and damage estimation – slower storms cause more damage

III. INVENTORY MODULEA. Building Databases

- 1. Government and private sources provide information



2. Data includes number of properties and values separated by
  - a. Line of business – residential, commercial, or industrial
  - b. Coverage – building, appurtenances, contents, and loss of use
  - c. Occupancy type – key factor in determining contents damage
  - d. Construction type – main factor in determining building damage

B. Gathering Information

1. Client can provide important details
2. Site-specific analysis may be appropriate

IV. VULNERABILITY MODULE

A. Overview

1. Two approaches for linking physical events and expected levels of damage
  - a. Engineering judgment
  - b. Building response analyses
2. Engineering judgment approach
  - a. Combine various expert opinions
  - b. Not easily updated for new information
3. Building response analyses
  - a. Focus on individual buildings and thus need to modify for portfolios
  - b. Division of buildings into a large number of building classes and subclasses
  - c. Steps in the analysis
    - 1) Identification of a typical building for a class
    - 2) Evaluation of building performance
  - d. Assumption that performance for a typical building applies to all buildings in the class
  - e. Derive EP curve for a large number of properties in the class

B. Identification of Typical Buildings

1. Need to identify as many classes as practical to represent structures in a region
2. Factors to consider
  - a. Homogeneity of structures within a class
  - b. Unique construction types in a region
  - c. Adoption and enforcement of building codes
  - d. Construction practices
3. Factors affecting structural responses define building classes
  - a. Building material
  - b. Structural system, e.g., moment framed
  - c. Height

4. Subdivision of building class by secondary modifiers, e.g., roof
- C. Evaluation of Building Performance
1. Defined by the relationship between the following:
    - a. Intensity of imposed force
    - b. Level of expected damage
  2. Damage from earthquakes is both structural and nonstructural
    - a. Use measures of building lateral response to predict damage
    - b. **Maximum interstory drift** – “ratio of the maximum relative lateral displacement of the two adjacent stories to the inter-story height”
  3. Damage from wind is mostly nonstructural and localized except in the following cases where roof damage can lead to a partial collapse
    - a. Mobile homes
    - b. Nonengineered buildings, such as wood frames
    - c. Buildings where the roof provides lateral stability
  4. Little test data on component or envelope resistances to wind
    - a. Most information comes from investigations of actual cases but are not always reliable
    - b. Wind tunnel tests are expensive and usually test high-quality structures
    - c. Lack of reliable wind recordings near the damaged structures
  5. Earthquake damage measured by fragility curves
    - a. **Fragility curve** – probabilities that a “specified damage state will be reached or exceeded as a function of the severity of ground motion at the site”
    - b. Categories of structural damage
      - 1) Minor damage
      - 2) Moderate damage
      - 3) Severe damage
      - 4) Collapse
    - c. Information can also be expressed by a damage function
      - 1) **Damage function** – “equation that relates the expected structural damage state of the entire building to the intensity of the event”
      - 2) Coefficient of variation describes uncertainty in the prediction of damage
      - 3) **Damage ratio** – “ratio of repair cost to replacement cost of the building” used to describe the amount of damage in a damage function
      - 4) Damage state described by the cumulative damage of the following:
        - a) Structural components, e.g., beams
        - b) Nonstructural components, e.g., cooling systems
        - c) Contents

- 5) Components affected by maximum deformation of the story where located, whereas contents affected by maximum floor acceleration
  - 6) Determine deformation and acceleration by damage surveys or lab tests using computer models
- d. Repair strategies vary with amount of damage from nothing to complete replacement

## V. LOSS MODULE

### A. Cost Models

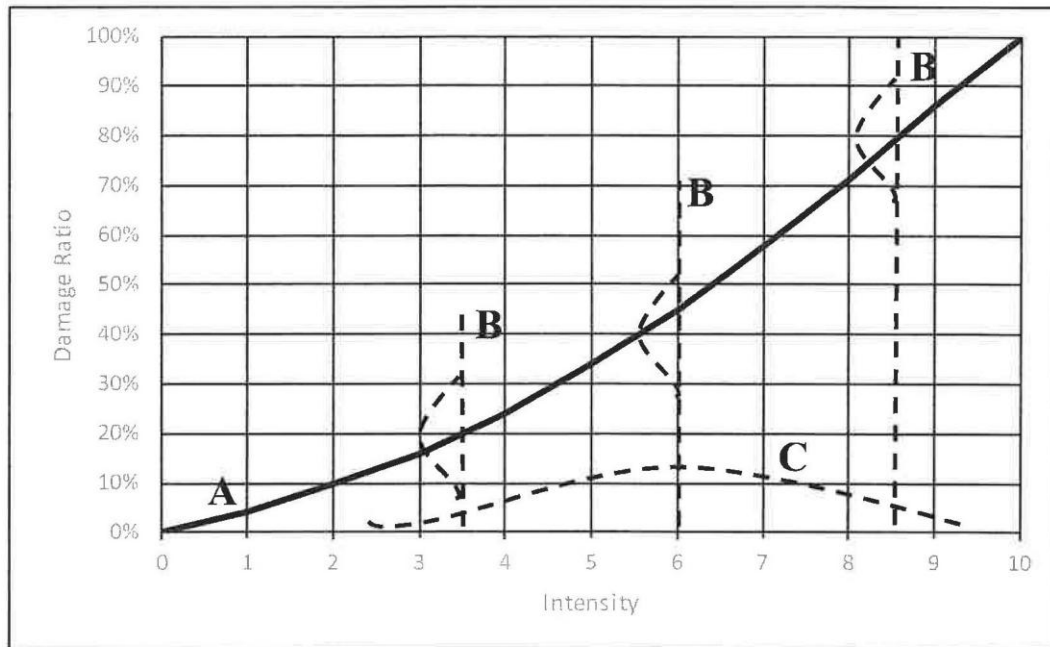
1. Convert estimates of physical damage into monetary loss
2. Various cost components of monetary loss
  - a. Repair of individual components
  - b. Inspection
  - c. Set up and debris removal

### B. Insured Losses

1. Apply the following policy conditions to total loss estimates:
  - a. Deductibles by coverage
  - b. Site-specific or blanket deductibles
  - c. Coverage limits and sublimits
  - d. Loss triggers
  - e. Coinsurance
  - f. Attachment points and limits for single- or multiple-location policies
  - g. Risk-specific reinsurance terms
2. Estimated insured losses checked against actual data
  - a. Available by zip code and line of business
  - b. Also frequently available by construction type and insurance coverage
3. Can express loss probabilities in terms of return periods, e.g., 5% equals a return period of twenty years

**PAST CAS EXAMINATION QUESTIONS**

1. An earthquake model produces the following damage function for a \$1,000,000 home in California:



The probability of an event occurring with a given intensity is:

| Intensity | Probability |
|-----------|-------------|
| 3.5       | 10%         |
| 6.0       | 5%          |
| 8.5       | 2%          |

- Identify what A, B, and C represent in the above graph.
- Determine the premium for this home given the following information:
  - The insurer's expense load is 20% of premium
  - The risk load is set to 8% of the standard deviation of the loss

(17-8-20-0.75/1.75)

1. a. A: the damage ratio at each intensity  
 B: the uncertainty in the damage ratio at the given intensities  
 C: the uncertainty in the felt intensity given a particular earthquake

b.

| Intensity           | Probability | Expected Loss |
|---------------------|-------------|---------------|
| 3.5                 | 10%         | 200,000       |
| 6.0                 | 5%          | 450,000       |
| 8.5                 | 2%          | 800,000       |
| Total Expected Loss |             | 58,500        |

$$SD = \sqrt{0.10 \times 200,000^2 + 0.05 \times 450,000^2 + 0.02 \times 800,000^2 - 58,500^2} = 153,306$$

$$\text{Risk Load} = 0.08 \times 153,306 = 12,264$$

$$\text{Premium} = \frac{58,400 + 12,264}{1.0 - 0.2} = 88,456$$

2. A catastrophe modeler would like to incorporate a new construction technique into a catastrophe model. This new technique would theoretically reduce the amount of building damage sustained during hurricane force winds. However, experts have not reached a consensus on the effectiveness of the new construction technique because it has not been exposed to an actual hurricane.
- a. Briefly describe which module(s) of the catastrophe model would need to be modified to account for the new information.
  - b. Classify the uncertainty created with the new construction technique as either aleatory or epistemic and briefly justify the selection.
  - c. Briefly describe and contrast two methods the modeler could use to incorporate uncertainty in this catastrophe model.

(18-8-17-0.5/0.5/1)

- 
2.     a.     It will impact both inventory module and vulnerability module. The insurer needs to update properties' information regarding this new construction technique in the inventory module and incorporate its susceptibility to loss damage in the vulnerability module.
- b.     Epistemic, because it is due to lack of data.
- c.     1)     Logic Tree assigns weight to parameter alternatives based on expert opinion. A weighted linear combination is calculated. This relies on simplified opinions but is easy to communicate.
- 2)     Simulation creates randomly sampled alternatives from a probability distribution of the parameter. It can handle complex situations but is difficult to compute