ACTEX Learning

Study Manual for Exam 7

4th Edition

Victoria Grossack, FCAS

A CAS Exam

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A. Estimation of Claims Liabilities

Introduction and Notes on Past Exam Questions and Answers and the Material

Greetings! In this actuarial study manual, you will find summary outlines and questions and answers for the readings for Part 7, the Advanced Estimation of Claims Liabilities.

Questions and parts of some solutions have been taken from material copyrighted by the Casualty Actuarial Society. They are reproduced in this study manual with the permission of the CAS solely to aid students studying for the actuarial exams. Some editing of questions has been done. Students may also request past exams directly from the society. I am very grateful to the CAS for its cooperation and permission to use this material. It is, of course, in no way responsible for the structure or accuracy of the manual.

Exam questions are identified by numbers in parentheses at the end of each question. CAS questions have four numbers separated by hyphens: the year of the exam, the number of the exam, the number of the question, and the points assigned.

In addition to the old exam questions and the summary outlines, review questions are included for most of the newer material. Some of the review questions are designed to help students process and memorize the material, while others have been designed to be more like potential exam questions.

Page numbers (p.) with solutions refer to the reading to which the question has been assigned unless otherwise noted. Note that parts of some exam questions may make use of material that is no longer included in the syllabus. Although I have made a conscientious effort to eliminate mistakes and incorrect answers, I am certain some remain. I encourage students who find errors to bring them to my attention. Please check our web site for corrections subsequent to publication.

♦ ♦ ♦

To the students who make use of this manual, feedback is welcome. Good luck! VAG

Introduction to Advanced Estimation of Claims Liabilities

Welcome to the study manual on the finer points of reserving! Note this manual has been prepared for the Spring 2025 CAS Exam 7. If you are trying to use this material for a different date, you may want to make sure that the syllabus has not changed.

The syllabus has 17 tasks you are supposed to master in order to pass the exam. They are:

Data Preparation, Organization & Analysis
A1. Perform data diagnostic analyses and adjust for data issues.

Unpaid Claim Point Estimates
A2. Calculate unpaid claims estimates.
A3. Test unpaid claims estimates for reasonableness.
A4. Estimate unpaid claims for various levels of coverage.
A5. Forecast premium reserves (e.g., reserves for retrospective premiums).

Unpaid Claim Stochastic Distributions
A6. Estimate parameters of unpaid claims distributions.
A7. Calculate the moments and percentiles of unpaid claims distributions.
A8. Simulate parameter percentiles and unpaid claims percentiles.
A9. Calculate the mean and prediction error of a reserve.
A10. Derive predictive distributions using stochastic methods.

Unpaid Claim Output & Diagnostic Analysis
A11. Test output of unpaid claims distributions for reasonableness.
A12. Test assumptions underlying reserving models.
A13. Develop a range of indicators.
A14. Calculate risk margins.

Reinsurance
A15. Adjust primary methods and data to be used for reinsurance reserving.
A16. Calculate ceded loss reserves.
A17. Describe the function and types of reinsurance.

Although the manual is organized by readings, we remind you of the tasks you are supposed to learn from each one.

Good luck in April/May, 2025!

Victoria Grossack

Eric Brosius
Loss Development Using Credibility

This reading addresses the following tasks listed in the syllabus:

A1. Perform data diagnostic analyses and adjust for data issues.
A2. Calculate unpaid claims estimates.
A3. Test unpaid claims estimates for reasonableness.
A6. Estimate parameters of unpaid claims distributions.
A11. Test output of unpaid claims distributions for reasonableness.

Keep these tasks in mind as you read the reading and review the outline!

Outline

I. Introduction

  A. What loss development method do you select when there are large random fluctuations in year to year loss experience?
  B. Least squares development is shown to provide the best linear approximation to the Bayesian estimate and is contrasted with other standard development techniques.

II. Notation

  A. $L(x)$- estimate of ultimate losses $\hat{y}$, given losses to date of x and historical experience (x_i, y_i)
  B. Y – random variable representing claims incurred
  C. X – random variable representing number of claims reported at year end
  D. $Q(X) = E(Y|X = x)$, expected total number of claims
  E. $R(X) = E(Y - X|X = x) = Q(X) - x$, expected number of claims outstanding
  F. MSE – mean squared error
  G. EVPV – Expected Value of the Process Variance - $E_Y(Var(X|Y))$
  H. VHM – Variance of the Hypothetical Means - $Var_Y(E(X|Y))$

III. Least Squares Development

  A. $L(x) = \text{a} + \text{b}x$, where
  B. $\text{b} = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2}$
  C. $\text{a} = \bar{y} - \text{b}\bar{x}$

IV. Special Cases of Least Squares Development

  A. When x and y are totally uncorrelated, b = 0
    1. $L(x) = \text{a}$, the "budgeted loss method"
  B. When the observed link ratios y/x are all equal, a = 0
    1. $L(x) = \text{b}x$, the "link ratio method"
  C. When b=1,
    1. $L(x) = \text{a} + x$, the "Bornhuetter-Ferguson method"

V. Hugh White's Question

A. If actual losses are higher than expected losses what do you do?
   1. Reduce the bulk reserve a corresponding amount (Budgeted Loss Method)
   2. Leave the bulk reserve at the same percentage level of expected losses (Bornhuetter-Ferguson Method)
   3. Increase the bulk reserve in proportion to the increase in actual reported over expected reported (Link Ratio Method)

B. These options are 3 points on the least squares continuum and the actual answer is likely to lie somewhere on that continuum.

VI. Bayesian Development Examples

A. Various examples using Bayesian estimation are used to show that the least squares estimate is superior to the link ratio, budgeted loss and Bornhuetter-Ferguson estimates:

B. Simple Model
   1. included to demonstrate method
   2. $Q(x) = \frac{2}{3}x + \frac{1}{3},\quad R(x) = -\frac{1}{3}x + \frac{1}{3}$, based on parameters in example
   3. The function $Q(x)$ does not align with any of the three special cases, but does lie on the least squares continuum.

C. Poisson-Binomial Example
   1. Poisson process determines ultimate claims (y) and reported claims (x) are determined by a Binomial process with the Poisson outcome y as the first parameter.
   2. $Q(x) = x + 2$, based on the parameters given in the paper
   3. This example is used to show that the link ratio method can't reproduce the Bayesian estimate Q(x), since there is no c, such that $cx \equiv x + 2$.
   4. Alternative Options for c
      a. Unbiased Estimate - $E\big((c-1)X\big) = \mu$
      b. Minimized MSE - minimize $E\big(\big((c-1)X - \mu\big)^2\big)$
      c. $c = E\left(\frac{Y}{X}\middle|X \neq 0\right)$
      d. Salzmann's Iceberg Technique - $d = E\left(\frac{X}{Y}\middle|Y \neq 0\right)$, $c = d^{-1}$

D. General Poisson-Binomial Case
   1. $Q(x) = x + \mu(1-d),\ R(x) = \mu(1-d)$
   2. This is consistent with the form of the Bornhuetter-Ferguson estimate.

E. Negative Binomial-Binomial Case
   1. $R(x) = \frac{(1-d)(1-p)}{1-(1-d)(1-p)}(x + r)$
   2. By plugging in sample parameter values it can be seen that the special cases of the least squares do not apply, but the result does lie on the least squares continuum.

F. Fixed Prior Case – the ultimate number of claims is known
   1. $Q(x) = k,\ R(x) = k - x$
   2. This is consistent with the budgeted loss method.

G. Fixed Reporting Case – percentage of claims reported at year end is always d
   1. $Q(x) = d^{-1}x,\ R(x) = (d^{-1} - 1)x$
   2. This is consistent with the link ratio method.

VII. The Linear Approximation – Development Formula 1

A. Pure Bayesian analysis requires significant knowledge about the loss and loss reporting process, which may not be available. A linear approximation can be used instead (Bayesian Credibility).
B. Development Formula 1 gives the best linear approximation to Q:
C. $L(x) = \left(x - E(X)\right)\frac{Cov(X,Y)}{Var(X)} + E(Y)$
D. With historical experience, we can estimate the parts:
E. $Cov(X,Y) = \overline{XY} - \bar{X}\bar{Y},\ Var(X) = \overline{X^2} - \bar{X}^2,\ E(X) = \bar{X},\ E(Y) = \bar{Y}$
F. Which gives the general least squares equation:
G. $L(x) = (x - \bar{X})\frac{\overline{XY} - \bar{X}\bar{Y}}{\overline{X^2} - \bar{X}^2} + \bar{Y}$
H. Potential problems in parameter estimation:
   1. Major changes in loss experience should be adjusted for:
      a. Inflation
      b. Exposure growth
   2. Sampling error
   3. Should substitute link ratio method when $a < 0$
   4. Should substitute budgeted loss method when $b < 0$

VIII. Credibility Form of the Development Formula – Development Formula 2

A. If there is a real number $d \neq 0$, such that $E(X|Y = y) = dy$ for all y, then the best linear approximation to Q is given by development formula 2:
B. $L(x) = Z\frac{x}{d} + (1 - Z)E(Y),\ where\ Z = \frac{VHM}{VHM + EVPV}$
C. This is a credibility weighting of the link ratio method and the budgeted loss method.
D. Special Cases:
   1. Poisson-Binomial and other Bornhuetter-Ferguson Cases
      a. $Z = d$
   2. Negative Binomial-Binomial Case
      a. $Z = \frac{d}{(d + p(1 - d))}$

IX. The Case Load Effect – Development Formula 3

A. If the rate of claim reporting is a decreasing function of the number of claims and there are real numbers $d \neq 0$ such that $E(X|Y = y) = dy + x_0$, then define development formula 3:

1. $L(x) = Z\frac{x - x_0}{d} + (1 - Z)E(Y)$

X. Mechanics of the Least Squares Approach

A. Adjust data for exposure growth and inflation
B. Develop most mature years to ultimate based on assumed tail factor
C. Develop next oldest year to ultimate using least squares on the complete years
D. Repeat one year at a time until all years have been developed

Past CAS Examination Questions

1. As the result of recent tort reform, general liability expected ultimate losses decreased from \$60 million to \$50 million for accident year 2005. Without the reform, 55% of ultimate accident year 2005 losses would have been reported within twelve months. With the reform, this percentage is expected to rise to 63%. At December 31, 2005, \$35 million of losses have been reported for accident year 2005.

   a. What is the link ratio estimate of the ultimate loss for accident year 2005?
   b. What is the Bornhuetter-Ferguson estimate of the ultimate loss for accident year 2005?
   c. Given that Y is expected ultimate losses and X is reported losses at 12 months, and using the estimates below, what is the ultimate loss for accident year 2005, using Brosius's Bayesian credibility method?

      $Var_Y[E(X|Y)] = 14.3$ $E_Y[Var(X|Y)] = 57$

   d. Why is it inappropriate to use the least-squares method in the situation described in this case? (06–6–15–.5/.5/1/.5)

2. An insurer has been experiencing a deteriorating loss ratio for the last five years on its personal auto business, due to the weakening of underwriting standards. Explain why the least-squares development method may not be appropriate. (07–6–42b–.5)

3. Given the following:

	Cumulative Reported Losses (\$000) Age of Development in Months			
Acc Year	12	24	36	48
2004	8,847	12,204	14,332	17,021
2005	10,280	14,650	16,807	
2006	11,747	14,826		
2007	12,077			

   a. Estimate the cumulative reported loss as of 24 months for accident year 2007 using the link ratio method.
   b. Estimate the cumulative reported loss as of 24 months for accident year 2007 using the budgeted loss method.
   c. Estimate the cumulative reported loss as of 24 months for accident year 2007 using the least-squares method. (08–6–9–.5/.5/1)

4. Given the following reported loss information:

Accident Year	As of 60 Months	As of 72 Months
2000	$40,000	$45,000
2001	30,000	60,000
2002	40,000	42,000
2003	30,000	32,000
2004	50,000	

a. Use Brosius' least-squares method to calculate the expected losses for accident year 2004 at 72 months.

b. Briefly explain whether least squares is an appropriate method to use in this situation. (09–6–3–2/.5)

5. Given the following information ($000):

Accident Year	Incurred Loss at 12 Months	Incurred Loss at 24 Months
2006	10,000	12,000
2007	16,000	20,000
2008	10,000	16,000
2009	15,000	

Use the method of least squares development to calculate the estimated incurred loss at 24 months for the accident year 2009. (10-6-11-2)

6. Given the following information ($000) for a line of business:

Accident Year	Written Premium	Earned Premium	Cumulative Reported Losses 12 Months	24 Months	36 Months
2007	5,756	4,779	413	2,310	5,845
2008	6,907	5,735	0	541	1,309
2009	8,289	6,882	936	2,311	
2010	9,946	8,258	50		

- The tail factor from 36 months to ultimate is 1.050.

a. Use the least squares method to estimate ultimate losses for the 2009 accident year.
b. Discuss the reasonability of the estimate derive in part a. above, relative to the estimate that would be produced by the link ratio method.
c. Illustrate graphically the relationships between the link ratio method, budgeted loss method and least squares method in modeling the loss development process. (11-7-1-1/0.5/1.5)

7. Given the following information:

	Incurred Loss Ratio	
Accident Year	As of 36 Months	As of 48 Months
2006	0.222	0.375
2007	0.451	0.675
2008	0.446	0.605
2009	0.228	

a. Estimate the loss ratio for accident year 2009 as of 48 months using the least squares method.
b. An alternate approach to estimating the accident year 2009 loss ratio as of 48 months is to use the arithmetic average of the link ratio method and the budgeted loss ratio method. Using the answer from part a. above, demonstrate whether this averaging approach is optimal. (12-7-4-1.5/1.5)

8. Given the following information:

Cumulative Losses ($000,000)		
Accident Year	Reported at 24 Months	Ultimate Loss
2008	12	18
2009	10	16
2010	14	20
2011	12	18
2012	21	

An insurer writes annual policies that incept on January 1. Exposure and coverage levels were constant for 2008 through 2011. On January 1, 2012, policy coverage was expanded and pricing actuaries estimated the following:

Loss amounts will increase by 25% due to the expanded coverage.
75% of ultimate losses are expected to be reported by 24 months, with a standard deviation of 8% of estimated ultimate loss.
Standard deviation of accident year 2012 ultimate loss will be $3 million.

a. Calculate the projected accident year 2012 ultimate loss using Bayesian credibility methods.

b. Explain why the least squares method is not appropriate for calculating the accident year 2012 loss.
(14-7-1-2:1.5/.5)

9. Given the following information ($000,000):

Accident Year	Cumulative Reported Loss @ 24 Months	Ultimate Loss
2011	36	75
2012	40	71
2013	35	64
2014	25	

a. Using the least-squares method, estimate ultimate loss for Accident Year 2014.

b. For each of the following scenarios, briefly describe a potential problem with the least-squares method:
i. The slope parameter is negative
ii. The intercept parameter is negative

c. Due to a regulatory change, the following is anticipated:
- No change in the reporting pattern
- Standard deviation of reported loss as of 24 months will be 10% of estimated ultimate loss
- Expected ultimate loss for 2014 will decrease 20%
- Standard deviation of accident year 2014 ultimate loss is expected for be $6,000,000

Using the Bayesian credibility method, estimate the revised ultimate loss for accident year 2014.
(16-7-2-3.25:1.25/0.5/1.5)

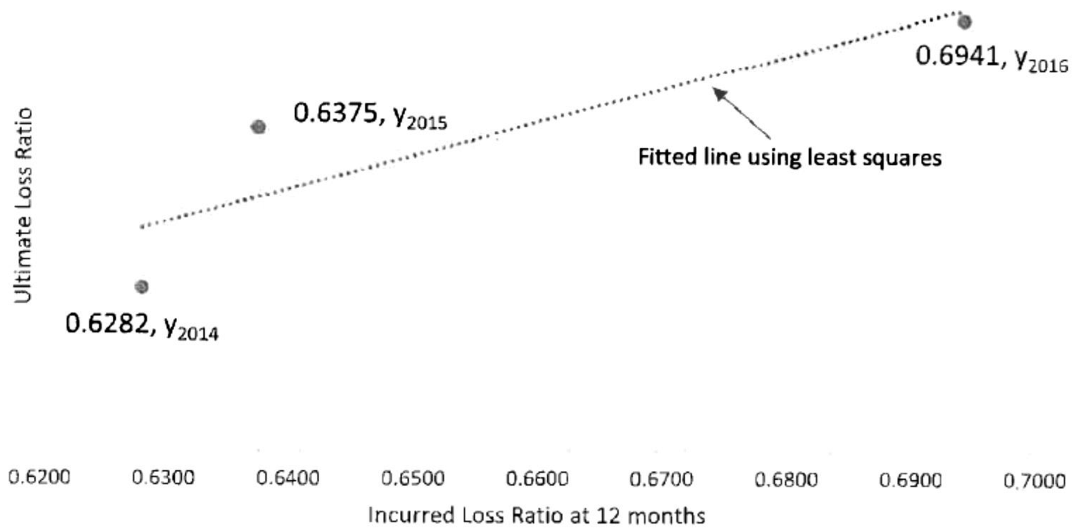
10. Given the following loss ratio triangle:

	Cumulative Reported Loss Ratios				
Accident Year	12 months	24 months	36 months	48 months	60 months
2010	3.0%	10.0%	15.7%	37.0%	37.0%
2011	5.1%	5.1%	25.0%	44.2%	48.0%
2012	2.5%	3.0%	40.0%	57.0%	59.2%
2013	1.6%	15.7%	22.2%	21.0%	
2014	0.0%	7.8%	16.7%		
2015	6.3%	12.4%			
2016	4.7%				

Assume a tail factor of 1.15 from 60 months to ultimate

Calculate the accident year 2014 ultimate loss ratio using the least squares method.
(17-7-2-1.75)

11. Given the following information:



- The equation for the line in the graph above is y = 0.1126 + bx.
- $\overline{xy} = 0.5979$
- Cumulative incurred loss for accident year 2017 at 12 months is $6,000,000.
- Earned premium for accident year 2017 is $8,000,000.
- Assume there is no further development after 24 months.

Calculate the ultimate loss for accident year 2017 using the least squares method. (18-7-4-2)

12. An insurance company historically never attempted to recover salvage and subgrogation ("S&S") on claims. On January 1, 2018, the insurer enters into a one-year agreement with an S&S recovery vendor, requiring the vendor to pursue all S&S opportunities for accident year 2018.

The insurance company's IT department generated the following loss development triangles as of December 31, 2018 (assume no development after 36 months):

Incremental Paid Loss Gross of S&S ($000s) as of (months)					Incremental Paid Loss Net of S&S ($000s) as of (months)			
Acc. Year	12	24	36		Acc. Year	12	24	36
2016	16,500	6,000	2,500		2016	16,500	6,000	2,500
2017	17,000	5,000			2017	17,000	5,000	
2018	14,000				2018	11,000		

Prior to 2018, the Actuarial department estimated unpaid losses using the following Bayesian model:

- C_{ij} represents the incremental losses for accident year i as of development year j which follow an overdispersed Poisson ("ODP") distribution with mean $x_i y_j$ and variance $\phi\, x_i y_j$.
- x_i represents the expected ultimate losses for accident year i.
- y_j represents the proportion of ultimate losses that emerge in development year j.

- The prior distribution for xi is gamma with mean α_i/β_i and variance α_i/β_i^2.
- ϕ represents the dispersion parameter for the ODP distribution, which is 93.
- λ_j represents the incremental chain ladder loss development factor for development year j.
- D_{ij} represents the cumulative losses for accident year as of development year j.
- M_i represents the value for ultimate losses for accident year I that is obtained using expert knowledge about the losses.
- The mean of C_{ij} for this Bayesian model is

  $Z_{ij}(\lambda_j - 1)D_{i,j-1} + (1 - Z_{ij})(\lambda_j - 1)M_i \frac{1}{\lambda_j \lambda_{j+1} \dots \lambda_n}$, where $Z_{ij} = \frac{\sum_{k=1}^{j-1} y_k}{\beta_i \varphi + \sum_{k=1}^{j-1} y_k}$.

  The Actuarial department believes that the S&S recovery vendor agreement has resulted in a slowdown of the gross loss payment pattern for calendar year 2018 and will continue in calendar years 2019 and 2020.

  To estimate the total unpaid losses gross of S&S for accident year 2018, the Actuarial department is considering the following prior gamma distributions for the Bayesian model above:

Parameter	Option 1	Option 2	Option 3
α	50	200	1,250
β	0.002	0.008	0.100

To estimate the total S&S recoveries for accident year 2018, the Actuarial department analyzed S&S data from competitors of comparable size to find:

- The expected S&S recoveries per accident year were $5M.
- The standard deviation of the S&S recoveries was $1M.
- The expected percent of the S&S recoveries through 12 months was 75%.
- The standard deviation of the percent of S&S recoveries received through 12 months was 10%.

a. Select the prior gamma distribution most appropriate to address the concern that the vendor agreement has slowed the gross payment pattern. Justify the selection.

b. Calculate the incremental losses gross of S&S for accident year 2018 expected to emerge between 12 and 36 months with the most appropriate prior distribution from part a. above.

c. Describe model risk with respect to the Bayesian model for incremental losses gross of S&S.

d. Calculate the total S&S recoveries for accident year 2018 using Bayesian credibility.

e. Describe estimation risk with respect to the Bayesian model for S&S recoveries.

f. Calculate the total unpaid losses net of S&S for accident year 2018.

g. Describe two types of operational risk introduced by the vendor agreement and recommend unique key risk indicator ("KRI") to monitor each risk. (7-19S-1-7:1/2/0.5/1.5/0.5/0.5/1)

Solutions to Past CAS Examination Questions

1. a. $x/d = 35\overline{M}/.63 = 55{,}555{,}556$, p. 2.

 b. $L = 35\overline{M} + dE[Y] = 35\overline{M} + (.37)(50\overline{M}) = 53.5\overline{M}$, p. 3.

 c. $Z = VHM/(VHM + EVPV) = 14.3/(14.3 + 57) = .201$

 $L(x) = Zx/d + (1 - Z)E[Y] = (.201)(55{,}555{,}556) + (1 - .201)(50\overline{M}) = 51{,}116{,}667$, pp. 13–15.

 d. It is inappropriate because there are significant changes in the loss history, p. 19.

2. It is not appropriate when "year to year changes are due largely to systematic shifts in the book of business," pp. 12, 19.

3. a. $\bar{x} = (8{,}847 + 10{,}280 + 11{,}747)/3 = 10{,}291$

 $\bar{y} = (12{,}204 + 14{,}650 + 14{,}826)/3 = 13{,}893$

 $c = \bar{y}/\bar{x} = 13{,}893/10{,}291 = 1.35$ $L(x) = cx = (1.35)(12{,}077) = 16{,}304$

 b. $L(x) = \bar{y} = 13{,}893$

 c. $\overline{xy} = [(8{,}847)(12{,}204) + (10{,}280)(14{,}650) + (11{,}747)(14{,}826)]/3 = 144{,}243{,}937$

 $\overline{x^2} = [(8{,}847)^2 + (10{,}280)^2 + (11{,}747)^2]/3 = 107{,}313{,}273$

$$b = \frac{\overline{xy} - (\bar{x})(\bar{y})}{\overline{x^2} - (\bar{x})^2} = \frac{144{,}243{,}937 - (10{,}291)(13{,}893)}{107{,}313{,}273 - 10{,}291^2} = .902$$

 $a = \bar{y} - b\bar{x} = 13{,}893 - (.902)(10{,}291) = 4{,}611$

 $L(x) = a + bx = 4{,}611 + (.902)(12{,}077) = 15{,}504$, pp. 2–3.

4. a. $\bar{x} = (40{,}000 + 30{,}000 + 40{,}000 + 30{,}000)/4 = 35{,}000$

 $\bar{y} = (45{,}000 + 60{,}000 + 42{,}000 + 32{,}000)/4 = 44{,}750$

 $\overline{xy} = [(40{,}000)(45{,}000) + (30{,}000)(60{,}000) + (40{,}000)(42{,}000) + (30{,}000)(32{,}000)]/4$

 $\overline{xy} = 1{,}560\overline{M}$

$\overline{x^2}$ = $[(40{,}000)^2 + (30{,}000)^2 + (40{,}000)^2 + (30{,}000)^2]/4$ = $1{,}250\bar{M}$

$$b = \frac{\overline{xy} - (\bar{x})(\bar{y})}{\overline{x^2} - (\bar{x})^2} = \frac{1{,}560M - (35{,}000)(44{,}750)}{1{,}250M - 35{,}000^2} = -0.25$$

$a = \bar{y} - b\,\bar{x} = 44{,}750 - (-.25)(35{,}000) = 53{,}500$

$L(x) = a + bx = 53{,}500 + (-.25)(50{,}000) = 41{,}000$

b. Since b < 0, the least-squares estimate is not appropriate. Because of this the estimate produced by the budgeted loss method ($\bar{y}$ = 44,750) may be substituted, pp. 2–4.

5. $\bar{x} = (10{,}000 + 16{,}000 + 10{,}000)/3 = 12{,}000$
$\bar{y} = (12{,}000 + 20{,}000 + 16{,}000)/3 = 16{,}000$
$\overline{xy} = [(10{,}000)(12{,}000) + (16{,}000)(20{,}000) + (10{,}000)(16{,}000)]/3 = 200{,}000{,}000$
$\overline{x^2} = [(10{,}000)^2 + (16{,}000)^2 + (10{,}000)^2]/3 = 152{,}000{,}000$

$$b = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2} = \frac{200M - (12{,}000)(16{,}000)}{152M - (12{,}000)^2} = 1$$

$$a = \bar{y} - b\bar{x} = 16{,}000 - 12{,}000 = 4{,}000$$

$$L(x) = a + bx = 4{,}000 + 15{,}000 = 19{,}000$$

6. a. Ultimate losses for AY 2007 and 2008:
2007: $Ult = 5{,}845(1.05) = 6{,}137.25$
2008: $Ult = 1{,}309(1.05) = 1{,}374.45$

Loss ratios for AY 2007 and 2008 (Divide by earned premium):

Year	24 Months	36 Months	Ultimate
2007	48.3%	122.3%	128.4%
2008	9.4%	22.8%	24.0%
2009	33.6%		

$$\bar{x} = (0.483 + 0.094)/2 = 0.289$$

$$\bar{y} = (1.284 + 0.240)/2 = 0.762$$

$$\overline{xy} = [(0.483)(1.284) + (0.094)(0.240)]/2 = 0.321$$

$$\overline{x^2} = [(0.483)^2 + (0.094)^2]/2 = 0.121$$

$$b = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2} = \frac{0.321 - (0.289)(0.762)}{0.121 - (0.289)(0.289)} = 2.689$$

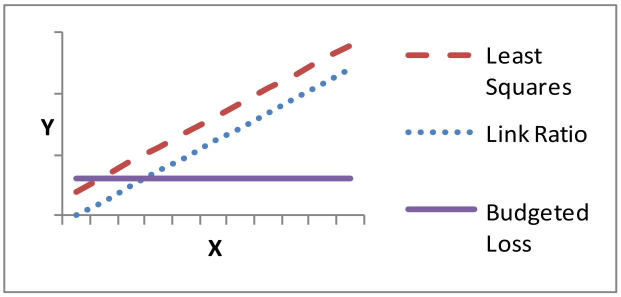
$$a = \bar{y} - b\bar{x} = 0.762 - 2.689(0.289) = -0.015$$

$L(x) = a + bx = -0.015 + 2.689(0.336) = 0.889$

$Ult\ Loss\ 2009 = 6{,}882(0.889) = 6{,}118.10$

b. Since the estimate of a is less than 0 the least squares method will produce estimates of y that are less than 0 when x is small. Brosius suggests substituting the link-ratio method when a < 0. The link-ratio method will produce positive estimates of y even for small values of x.

c.



7. a. $L(x) = a + bx$

$\bar{x} = (0.222 + 0.451 + 0.446)/3 = 0.373$

$\bar{y} = (0.375 + 0.675 + 0.605)/3 = 0.552$

$\overline{xy} = [(0.222)(0.375) + (0.451)(0.675) + (0.446)(0.605)]/3 = 0.219$

$\overline{x^2} = [(0.222)^2 + (0.451)^2 + (0.446)^2]/3 = 0.151$

$b = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2} = \frac{0.219 - (0.373)(0.552)}{0.151 - (0.373)(0.373)} = 1.104$

$a = \bar{y} - b\bar{x} = 0.552 - 1.104(0.373) = 0.140$

$L(x) = a + bx = 0.140 + 1.104(0.228) = 0.392$

$Ult\ Loss\ Ratio\ 2009 = 39.2\%$

b. In a credibility weighting $Z = b/c$, where $c = \bar{y}/\bar{x}$
$Z = 1.104/(0.552/0.373) = 0.746$
Since $Z = 0.746 \neq 0.5$ the arithmetic average does not produce an optimal solution.

8. a. X= loss reported at 24 months

Y= Ultimate losses

L(x)=Z(x/d)+(1 - Z)E[Y]
Z=VHM/(VHM+EVPV)
VHM =(E[D] × σ (y))^2 = ((.75)(3))^2 =5.0625
EVPV=Var(D)[Var(y) +E[y] ^{2}]=(0.08)^2 [3^2+[(1.25)({18+16+20+18}/4)]^2]=3.2976

Z=5.0625/(5.0625+3.2976)=.606

L(x)=(.606)(21/.75)+(1-.606)(22.5)=25.833 million

b. The least squares method is appropriate when the distribution of loss is not changing year over year. Given the coverage expansion and change in 2012 loss distribution, we cannot use the least squares method.

9. a.

$$\overline{X} = \frac{36+40+35}{3} = 37$$

$$\overline{Y} = \frac{75+71+64}{3} = 70$$

$$\overline{XY} = \frac{36\times75+40\times71+35\times64}{3} = 2593.33$$

$$\overline{X^2} = \frac{36^2+40^2+35^2}{3} = 1373.67$$

$$b = \frac{\overline{XY}-\overline{X}\,\overline{Y}}{\overline{X^2}-\overline{X}^2} = 0.713$$

$$a = \overline{Y} - b\times\overline{X} = 43.62$$

2014 Ultimate Loss = a + b × 25 = 61.45

b. i. If b < 0, then y decreases as x increases.
ii. If a < 0, then y is negative for small values of x.

c.

$$\sigma_d = 0.1$$

$$Y = 0.8\times70 = 56$$

$$\sigma_Y = 6$$

$$d = \frac{37}{70} = 0.5286$$

$$VHM = \sigma_Y^2 d^2 = 6^2(0.5286)^2 = 10.058$$

$$EVPV = \sigma_d^2\left[\sigma_Y^2 + Y^2\right] = (0.1)^2\left(6^2+56^2\right) = 31.72$$

$$Z = \frac{VHM}{VHM+EVPV} = \frac{10.058}{10.058+31.72} = 0.2407$$

$$L = 0.2407\left(\frac{0.25}{0.5286}\right) + \left(1-0.2407\right)\left(56\right) = 53.904$$

10. Need 2013 ultimate first:

$\overline{X}$ = 1/3 × (0.37+0.442+0.57) = 0.4607
$\overline{Y}$ = 1/3 ×1.15 × (0.37+0.48+0.592) = 0.5528
$\overline{XY}$ = 1/3 × (0.37 × 1.15 × 0.37+...) = 0.2632
$\overline{X}^2$ = 1/3 × (0.37^2+...) = 0.2191
b = ($\overline{XY}$ - $\overline{X}$ × $\overline{Y}$) / ($\overline{X}^2$ bar - ($\overline{X}$ bar) ^2)) = 1.2435
a = $\overline{Y}$ - b × $\overline{X}$ = -0.0201
Since a < 0, using link ratio method instead
2013 ultimate = 0.21 × 1.15 × (0.37+0.48+0.592)/(0.37+0.442+0.57) = 0.2520

Calculate 2014 ultimate
$\overline{X}$ = ¼ × (0.157+0.25+0.4+0.222) = 0.2573
$\overline{Y}$ = ¼ × (0.37 × 1.15+0.48 ×1.15+0.592 × 1.15+0.2520) = 0.4776
$\overline{XY}$ = ¼ × (0.157 × (0.37 × 1.15)+...) = 0.1333
$\overline{X}^2$ = ¼ × (0.157^2+...) = 0.0741
b = ($\overline{XY}$ - $\overline{X}$ × $\overline{Y}$) / ($\overline{X}^2$ - ($\overline{X}$) ^2)) = 1.3187
a = $\overline{Y}$ - b × $\overline{X}$ = 0.1383
2014 ultimate = a + b × 0.167 = 0.359

11. $\bar{x} = \frac{0.6282+0.6375+0.6491}{3} = 0.6533$

$\bar{x}^2 = 0.4268$

$\overline{x^2} = \frac{0.6282^2+0.6375^2+0.6491^2}{3} = 0.4276$

$\bar{y} = 0.1126 + 0.6533b$

$b = (\overline{xy} - \bar{x}\bar{y})/(\overline{x^2} - \bar{x}^2) = (0.5979 - 0.6533\bar{y})/(0.4276 - 0.4268)$

$b = [0.5979 - 0.6533 \times (0.1126 + 0.6533b)] \div 0.0008 = 1.2261$

For AY 2017:

$$y = 0.1126 + 1.2261x = 0.1126 + 1.2261 \times \frac{6{,}000{,}000}{8{,}000{,}000} = 1.0322$$

$$Ultimate = 1.0322 \times 8{,}000{,}000 = 8{,}257{,}600$$

12. a. Mean (α/β):
Option 1: 25,000
Option 2: 25,000
Option 3: 12,500
Variance:
Option 1: 12,500,000
Option 2: 3,125,000
Option 3: 125,000
Select Option 2. Mean value is more reasonable. Variance is smaller which gives less credibility to LDF, and more credibility to prior estimate.

b. Calculated LDFs:

12-24 LDF = (22500 + 22000)/(16500 + 17000) = 1.3284
24-36 LDF = 25000/22500 = 1.111
LDF12 = 1.111 × 1.328 = 1.475, %paid12 = 1/1.475 = 0.678
LDF24 = 1.111, %paid12 = 1/1.111 = 0.900
Calculate Z12 = 0.678/(0.008 × 93 + 0.678) = 0.477
Z24 = 0.900/(0.008 × 93 + 0.900) = 0.547
E[incremental loss at 24 mo] = 0.477 × 14000 × (1.328-1) + (1-0.477) × 25000 × (0.9-0.68) = 5093
E[incremental loss at 36 mo] = 0.547 × (1.111-1) × (14000 + 5093) + (1-0.547) × 25000 × (1-0.9) = 2291.77
Total expected emergence for AY 2018 = 5093 + 2291.77 = **7384.77** (in 000s)

c. Model risk is risk of not specifying the correct model. In Bayesian model, we weigh our faith in the specified model via the β parameter, which controls how much weight is given to the model (chain ladder) versus our a priori estimate. Higher β reduces the credibility measure, thus giving more weight to our a priori estimate.

d. VHM = Var [E(X/Y)] = d2 · 2 = (0.75) 2 x 1 2 = 0.563
EVPV = E [Var (X/Y)] = σd2 (σy2 + E(Y)2 = (0.1)2 x (12 + 52) = 0.26 Z = 0.563 / (0.653 + 0.26) = 68.4%
UltSS = 68.4% x [(14 – 11) / 0.75] + (1 – 68.4%) x 5 = **4.31 million**

e. Estimation risk is the risk that the forms and parameters chosen don't reflect the "true" form and parameters, due to estimating from a sample of the data.
For S&S, the department took data from competitors of comparable size. However, there could be a difference in their mix of business, their credibility of data, and assumptions/definitions that can cause our estimates from it to misrepresent the data.

f. Future Expected Recoveries = 4.316 – 3M = 1.316M
Total Unpaid Net of S&S = 7.396M – 1.316M = 6.08M

g. Two of the following:

- Clients & Business Practices – not meeting professional obligations to clients; could be trying to over recover and receiving complaints to the DOI. (KRI – monitor number of complaints)
- External Fraud – S&S company may put falsified information into the claims and return less money to the company (KRI – external auditor's reports)
- Execution & Process Change Management – employees have never done S&S before; this could lead to people getting upset over the added work or over glitches that may occur during implementation (KRI – monitor employee turnover ratio)

- Internal Fraud – since there will be an increased number of transactions with another company, there may be more opportunities for accounting people to slip in fraudulent transactions to steal money from the insurer (KRI – keep track of the number of ledger entries for transactions with the vendor and look for any unexpected increases)
- Recovery Risk – comes from uncertainty of recovery which could be a significant amount of total gross claim (KRI – recovery amount / total gross claim amount)
- System Failure – integrating our systems within the new S&S processes could cause problems, perhaps from tying our reporting systems & the vendors payments, and tracking systems; this could possibly cause processing center down time (KRI – processing center down time)
  Note that problem and answers draw on several readings.

Thomas Mack
Credible Claims Reserves: The Benktander Method

This reading addresses the following tasks listed in the syllabus:

A1. Perform data diagnostic analyses and adjust for data issues.
A2. Calculate unpaid claims estimates.
A3. Test unpaid claims estimates for reasonableness.
A9. Calculate the mean and prediction error of a reserve.
A11. Test output of unpaid claims distributions for reasonableness.
A12. Test assumptions underlying reserving models.

Keep these tasks in mind as you read the reading and review the outline!

Outline

I. Introduction

A. Mack reviews the Benktander method, which is a credibility weighting of the chain ladder and Bornhuetter-Ferguson techniques
B. The paper focuses on the development of notation for a single accident year

II. Notation

A. R_{BF} – Bornhuetter-Ferguson reserve estimate
B. U_{BF} – Bornhuetter-Ferguson estimate ultimate losses
C. R_{CL} – Chain Ladder reserve estimate
D. U_{CL} – Chain Ladder estimate of ultimate losses
E. R_{GB} – Benktander reserve estimate
F. U_{GB} – Benktander estimate of ultimate losses
G. U_0 – A priori estimate of ultimate claims
H. C_k – Claims amount paid up to time k
I. p_k – Proportion of ultimate claims that are expected to be paid by time k

III. Review of the Established Methods

A. Bornhuetter-Ferguson
   1. $R_{BF} = q_k U_0$, where $q_k = 1 - p_k$
   2. $U_{BF} = C_k + R_{BF}$
   3. This is the standard B-F derivation – the reserve is determined independently of the losses to date
B. Chain Ladder
   1. $U_{CL} = \frac{C_k}{p_k}$
   2. $R_{CL} = U_{CL} - C_k = q_k U_{CL}$
   3. Chain ladder considers the claims paid to date to be fully credible and ignores the a priori estimate
C. The Bornhuetter-Ferguson and Chain Ladder methods are extreme cases of a credibility mixture.
   1. B-F gives 0% credibility to actual losses
   2. CL gives 100% credibility to actual losses
   3. The Benktander method seeks to find a credibility mixture that adjusts for the credibility of actual losses

IV. The Benktander Method

A. Benktander proposed to take the B-F reserve formula, $R_{BF} = q_k U_0$, and replace U_0 with $U_{p_k} = p_k U_{CL} + (1 - p_k) U_0$
   1. Notice that $U_{p_k} = C_k + q_k U_0 = U_{BF}$

B. $R_{GB} = q_k U_{BF}$
   1. This is why the Benktander method can be referred to as the iterative B-F.

C. $U_{GB} = C_k + R_{GB} = \left(1 - q_k^2\right) U_{CL} + q_k^2 U_0$, after manipulation

V. General Iteration Formula

A. The iteration formula can be defined generally as:

$$R^{(m)} = q_k U^{(m)}, \qquad U^{(m+1)} = C_k + R^{(m)}$$

B. Which gives the credibility mixture:

$$U^{(m)} = (1 - q_k^m) U_{CL} + q_k^m U_0, \qquad R^{(m)} = (1 - q_k^m) R_{CL} + q_k^m R_{BF}$$

   1. m = 0; Initial reserve
   2. m = 1; B-F reserve
   3. m = 2; GB reserve
   4. m = ∞; CL reserve

VI. Neuhaus' Analysis

A. In the credibility weighting of the chain ladder and Bornhuetter-Ferguson reserves, $R_c = cR_{CL} + (1 - c)R_{BF}$, Neuhaus defined c^* to be the weighting that minimizes MSE(R_c).

B. The mean squared error of the Benktander reserve is almost as small as the optimal credibility reserve
   1. Except when p_k is small and c^* is large

C. Benktander has a smaller mean squared error than B-F whenever $c^* > \frac{p_k}{2}$

Past CAS Examination Questions

1. You are given the following information:

Accident Year	Earned Premium	Expected Loss Ratio	Paid Age to Ultimate Factor	Paid Loss as of Dec 31, 2003
2002	$25,000	65%	2.30	$7,200
2003	15,000	75%	4.00	3,375

Use the Benktander method described by Mack to estimate the ultimate losses (in Mack's notation, $U^{(2)}$ or U_{GB}) for accident years 2002 and 2003. Show all work. (04–6–31–2)

2. You are given the following information:

   i) Paid loss as of December 31, 2004 is $6,000,000.
   ii) The proportion of ultimate loss expected to be paid as of December 31, 2004 is 30%.
   iii) The *a priori* ultimate loss estimate is $16,000,000.

   Calculate the Benktander ultimate loss as of December 31, 2004. (05–6–16–1.5)

3. Given the following information:

Prior ultimate loss estimate	$9,000,000	Paid losses	$8,000.000
Loss development factor	1.250		

   a. Calculate the loss reserve using the Benktander method.
   b. Explain why the Benktander method generally represents an improvement over the Bornhuetter-Ferguson method.
   c. Explain why the Benktander method generally represents an improvement over the chain ladder method. (06–6–16–1/.5/.5)

4. Given the following for an accident year:

   Earned premium $20,000,000 Reported losses as of 12 months $10,000,000
   Expected loss ratio 70% Coefficient of variation of the loss ratio .70
   Coefficient of variation of percent of loss reported .45

   The expected reporting pattern is as follows:

Age in months	12	24	36	48	60
Percent reported	40%	60%	80%	90%	100%

   The mean and coefficient of variation of the reporting pattern are independent of the ultimate losses.

   a. Calculate the linear approximation to the Bayesian credibility estimate as of twelve months of ultimate loss for this accident year.

b. At age twelve months, determine which of the following methods produces the estimate of ultimate loss closest to the Bayesian credibility estimate determined in a.

i) Chain ladder ii) Bornhuetter-Ferguson iii) Benktander.

c. Explain how the Benktander formula can be described as a credibility-weighted average. (08–6–10–1.5/1/.75)

5. Given the following information for accident year 2011 as of December 31, 2011:

- Accident year 2011 paid loss: $700,000
- 2011 earned premium: $3,000,000
- Initial expected loss ratio: 62.5%
- 12-24 month paid link ratio: 1.50
- 12-ultimate cumulative paid LDF: 2.50

a. Calculate accident year 2011 ultimate loss estimates as of December 31, 2011 using each of the following three methods:

i. Chain ladder
ii. Bornhuetter-Ferguson
iii. Benktander

b. Determine the accident year 2011 incremental paid loss in 2012 that would result in the Benktander ultimate loss estimate being $50,000 greater than the Bornhuetter-Ferguson ultimate loss estimate for accident year 2011, as of December 31, 2012. Assume all selected development factors remain the same. (12-7-1-1.25/1.5)

6. Given the following information:

Calculated Ultimate Losses ($000)

Accident Year	Born-Ferg Ultimate	Benkt. Ultimate
2009	12,181	12,181
2010	11,246	11,316
2011	8,428	8,204
2012	10,403	10,609

a. Calculate the 24-month-to-ultimate cumulative development factor that would result in the ultimate loss estimates shown above.
b. For accident year 2011, suppose that the Bornhuetter-Ferguson method is performed over multiple iterations. Deduce the ultimate loss estimate that will be produced as the number of iterations approaches infinity. (13-7-4-1.5/0.5)

7. Given the following information:

	Cumulative Loss Payments		
Accident Year	12 months	24 months	36 months
2013	1500	2700	3450
2014	1600	2740	
2015	1700		

- Exposures and premium are constant across all accidents year
- There is no development past 36 months

a. Calculate the total reserve indication as of December 31, 2015 using loss ratio factors and the Benktander method.

b. Calculate the fifth-iteration Benktander reserve method indication for accident year 2015.

c. Assuming $Var[U_i] = Var[U_i^{BC}]$, use Hürlimann's method for optimal credibility and minimum variance to calculate the reserve indication for accident year 2015.
(16-7-1-3.5:2/0.75/0.75)

8. Given the following information about accident year 2017 as of December 31, 2017.

Accident year 2017 paid loss = $850,000.
2017 earned premium = $4,000,000.
Initial expected loss ratio =67.5%.
12-24 month incremental paid link ratio = 1.60.
12-ultimate cumulative paid LDF = 3.00.

a. Determine the accident year 2017 incremental paid loss in 2018 that would result in the Benktander ultimate loss estimate being $100,000 less than the Bornhuetter-Ferguson ultimate loss estimate for accident year 2017 as of December 31, 2018.

b. Briefly describe when the Benktander ultimate loss estimate would be greater than the Bornhuetter-Ferguson ultimate loss estimate as of December 31, 2018.

c. Explain why it may not be appropriate to use the Bornhuetter-Ferguson method when losses develop downward. (18-7-5-2.25:1.5/0.25/0.5)

9. Given the following information as at December 31, 2018.

Accident Year	Earned Premium ($000)	Incremental Paid Loss ($000) as of (months)		
		12	24	36
2016	5,000	1,800	700	500
2017	6,000	2,000	800	
2018	8,000	2,200		

Assume there is no further development after 36 months.
$Var[U_i] = Var[U_i^{BC}]$

a. Calculate the accident year 2018 Benktander reserve estimate (RGB).

b. Calculate the accident year 2018 optimal credible reserve estimate (Rc).

c. Identify which of Rc or RGB is the preferable reserve from a statistical point of view and briefly describe a supporting reason.

d. Describe the effect on the Benktander credibility for accident year 2018 if the incremental paid loss from 12 to 24 months for accident year 2017 was greater than the value in the table above.
(7-19S-2-3:1.5/0.5/0.5/0.5)

Solutions to Past CAS Examination Questions

1. U_{CL} = (Paid Losses)(ULDF)
$U_{CL/02}$ = (7,200)(2.30) = 16,560 $U_{CL/03}$ = (3,375)(4.00) = 13,500
U_0 = (Earned Premium)(ELR)
$U_{0/02}$ = (25,000)(.65) = 16,250 $U_{0/03}$ = (15,000)(.75) = 11,250
q_k = 1 – 1/ULDF $q_{k/02}$ = 1 – 1/2.30 = .565 q_k = 1 – 1/4 = .750

$$U_{GB} = \left(1 - q_k^2\right)U_{CL} + q_k^2 U_0$$

$U_{GB/02}$ = [1- (.565)^2][16,560] + (.565)^2(16,250) = 16,461
$U_{GB/03}$ = [1 - (.750)^2][13,500] + (.750)^2(11,250) = 12,234, pp. 334–35.

2. ULDF = 1/.3 = 10/3 q_k = 1 - 1/ULDF = 1 - 1/(10/3) = .7
U_{CL} = (Paid Losses)(ULDF) = (6 $\overline{M}$)(10/3) = 20 $\overline{M}$

$$U_{GB} = \left(1 - q_k^2\right)U_{CL} + q_k^2 U_0$$

U_{GB} = [1 - (.7)^2][20 $\overline{M}$] + (.7)^2(16 $\overline{M}$) = 18.04 $\overline{M}$, pp. 334–35.

3. a. q_k = 1 - 1/ULDF = 1 – 1/1.250 = .2
U_{BF} = $q_k U_0$ + Paid Losses = (.2)(9 $\overline{M}$) + 8 $\overline{M}$ = 9.8 $\overline{M}$
R_{GB} = $q_k U_{BF}$ = (.2)(9.8 $\overline{M}$) = 1.96 $\overline{M}$, pp. 334–35.

b. Unlike the Bornhuetter-Ferguson method, it gives some weight to paid losses and almost always has a smaller mean squared error, pp. 333–34.

c. Unlike the chain ladder method, it gives some weight to the initial estimate of ultimate losses and it almost always has a smaller mean squared error, pp. 333, 335.

4. a. $SD_{lossratio} = (CofV_{lossratio})(ELR) = (.70)(.70) = .49$

$VHM = (SD_{lossratio})^2(\text{Expected Percent Reported})^2 = (.49)^2(.40)^2 = .03842$

$EVPV = [SD_{lossesreported}]^2[(SD_{lossratio})^2 + (ELR)^2]$

$EVPV = [.18]^2 [(.49)^2 + (.70)^2] = .02366$

$k = EVPV/VHM = .02366/03842 = .6158$ $Z = n/(n + k) = 1/(1 + .6158) = .62$

$U_{CL} = 10\overline{M}/.4 = 25\ \overline{M}$ $U_0 = (\text{Earned Premium})(ELR) = (20\ \overline{M})(.70) = 14\ \overline{M}$

$U_B = ZU_{CL} + (1 - Z)(U_0) = (.62)(25\ \overline{M}) + (1 - .62)(14\overline{M}) = 20.82\ \overline{M}$, p. 337.

b. i) $U_{CL} = C_k/p_k = 10\ \overline{M}/.4 = 25\ \overline{M}$

ii) $q_k = 1 - p_k = 1 - .4 = .6$

$U_{BF} = q_k U_0 + C_k = (.6)(14\ \overline{M}) + 10\ \overline{M} = 18.4\ \overline{M}$

iii) $U_{GB} = \left(1 - q_k^2\right)U_{CL} + q_k^2 U_0 = [1 - (.6)^2][25\ \overline{M}] + (.6)^2(14\ \overline{M}) = 21.04\ \overline{M}$

The Benktander estimate is closest to the Bayesian credibility estimate, pp. 334–35.

c. It is a credibility-weighted average of the estimated ultimate claims amount without taking claims experience into account (U_0) and the chain ladder estimate (U_{CL}), p. 335.

5. a. Chain Ladder: $U_{CL} = 700{,}000 \times 2.50 = 1{,}750{,}000$

B-F: $U_{BF} = 700{,}000 + (3{,}000{,}000)\left(1 - \frac{1}{2.50}\right)(0.625) = 1{,}825{,}000$

Benktander: $U_{GB} = 700{,}000 + 1{,}825{,}000 \times \left(1 - \frac{1}{2.50}\right) = 1{,}795{,}000$

b. $U_{GB}^{2012} = (700{,}000 + x) + U_{BF}^{2012} \times \left(1 - \frac{1}{\left(\frac{2.5}{1.5}\right)}\right)$

$U_{BF}^{2012} = (700{,}000 + x) + (3{,}000{,}000) \times \left(1 - \frac{1}{\left(\frac{2.5}{1.5}\right)}\right)(0.625) = 1{,}450{,}000 + x$

$U_{GB}^{2012} = 1{,}280{,}000 + 1.4x$, after simplifying

$U_{GB}^{2012} = U_{BF}^{2012} + 50{,}000$

$1{,}280{,}000 + 1.4x = 1{,}450{,}000 + x$

$x = 550{,}000$

6. a. $R_{GB} = q_k \times U_{BF}$

$$(8204 - 7004) = \left(1 - \frac{1}{LDF_{24-Ult}}\right) \times 8428$$

$LDF_{24-Ult} = 1.166$ p. 335

b. Infinite iterations produces the chain-ladder estimate:

$R^{\infty} = R_{CL} = 1.166 \times 7004 = 8167$ p. 336

7. a. Assume premium = 5,000 for each accident year. (Other premium amounts may be assumed)

M_1 = [(1,500 + 1,600 + 1,700)/3 × 5,000] = 0.32
M_2 = [(1,200 + 1,140)/2 × 5,000] = 0.234
M_3 = 750/5,000 = .15
Expected loss ratio = 0.704. U^0 = .704 x 5,000 = 3,520
P_1 = 0.32/0.704 = 0.455; Q_1 = 1 - 0.455 = 0.545
P_2 = (0.32 + 0.234)/0.704 = 0.787; Q_1 = 1 - 0.787 = 0.213

2014_{ind} = 2,740/.787 x .213 = 742
2015_{ind} = 1,700/.455 x .545 = 2,036

2014_{coll} = 3,520 × .213 = 750
2015_{coll} = 3,520 × .545 = 1,918

2014_{bt} = 742 × .787 + 750 × .213 = 743
2015_{bt} = 2,036 × .455 + 1,918 × .545 = 1,972

Total reserve = 743 + 1,972 = 2,715

b. Expected Cost Reserves for AY 2015 2015_{Ec}= (5,000) × 70.4% - 1,700 = 1,820
Fifth Iteration Benktander Reserve = 2015_{ind} × (1- q^5) + 2015_{Ec} × q^5
= 2,036 × (1-0.545^5) + 1,820 × 0.545^5 = 2,025.6

c. Z = P_1 / (P_1 + √ P_1) = 0.455 / (0.455 + √0.455) = 0.403
Reserve = Z × 2015_{ind} + (1 – Z) × 2015_{coll} = 0.403 × 2,036 + 0.597 × 1,918 = 1,966

8. a. BF Ultimate – Benktander Ultimate = 100,000
BF Ultimate = 850 + *x* + 4,000 × (0.675) × [1 – (3.00/1.60)-1] = 2,110 + *x*
GB Ultimate = 850 + x + (2,110 + x) × [1 – (3.00/1.60)-1] = 1,834.67 + 1.467*x*
2,110 + *x* – 1,834.67 – 1.467*x* = 100
x = 375.45

b. Benktander ultimates would be greater than BF ultimates if the CL Ultimate > BF Ultimate, since Benktander is a weighting of the two methods.

c. When losses develop downward, the BF method will keep the forward looking IBNR the same, regardless of how losses to date have performed. Thus, the downward development will not affect IBNR. However, in reality, the downward development may be indicating salvage & subro trends that we would also want to apply to our IBNR. (Other answers possible)

9. a. $m_1 = \frac{1800+2000+2200}{5000+6000+8000} = 0.316$

$m_2 = \frac{700+800}{5000+6000} = 0.136$

$m_3 = \frac{500}{5000} = 0.1$

$ELR = 0.316 + 0.136 + 0.1 = 0.552$

$p_1 = \frac{0.316}{0.552};\ p_2 = \frac{0.316+0.136}{0.552} = 0.819;\ p_3 = 1$

$R^{GB} = q_k \times U^{BF}$

$U^{BF} = 2200 + (1-0.572) \times 0.552 \times 8000 = 4090$

$R^{GB} = (1-0.572) \times 4090 = 1750.52$

b. Since $Var(U_i) = Var(U_i^{BC}), Z^{opt} = \frac{p_i}{p_i+\sqrt{p_i}}\ because\ t_i = \sqrt{p_i}$

$R_c = Z^{opt} \times R^{Ind} + (1-Z^{opt}) \times R^{Coll}$

$R_c = 1646.7 \times 0.4307 + 1890.1 \times (1-0.4307) = 1785.74\ (in\ 000s)$

c. R_c is the preferable reserve because it minimizes the MSE of the reserve.

d. If that loss was greater, then the overall loss ratio would be larger. Since the incremental LR at 12 months stays the same, p would decrease, which means the credibility also decreases. If the incremental loss was greater than above, this would increase m_2, which would also increase the ELR. m_1 would stay the same so p2018 = m1/ELR would decrease. Since the credibility for Benktander is Z = p2018 the credibility would decrease as well.